Theory framework as a knowledge hub message #1

Presentation · October 2023

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October 2023

The goal

- Storing the theory of a particular subject area in one place and maintaining it (including formalization) through collective efforts.

- The concentration and verification of knowledge achieved in this case should give a powerful ordering of theoretical knowledge.

- Which will facilitate their formalization, i.e. mathematical notation, and therefore algorithmic processing.

We discuss what the framework of the theory is, intended for unified storage and collective accumulation of its results.

Introduction-1

Modern means of communication and data storage make it possible to keep the theory of a specific subject area in a single copy for all researchers in the world.

It is proposed to store only the most essential in theory in the frame:

-setting the structure for theory models,

-determination of the requirements that a structure must satisfy in order to be a model i.e. setting axioms,

-definitions of terms used in describing the subject area,

-determination of additional properties that all or some models have i.e. theorems and hypotheses,

-providing a proof of a hypothesis that makes it a theorem.

Introduction-2

Each element of the framework will contain formulations in several languages, both natural and formal.

All places where terms or identifiers corresponding to them are used contain a link to the block in which the corresponding term or identifier is entered.

The construction and maintenance of a theory framework is possible for both natural sciences and theoretical knowledge of engineering.

The elements of the framework (mainly definitions) will be embedded into all author's textbooks on this theory and articles using or developing the theory.

The frame elements will be assembled together for a particular task.

Structure-1

Every theory is a theory about something. In mathematics, these are usually structures of one kind or another. Let's consider an example of a structure for the theory of **undirected graphs**.

One of the basic components of structure, often encountered in mathematics, is a certain set - the *carrier* of the structure, which we will assume to be finite, and the other component will be some binary relation in it.

Such a structure, consisting of two components: **U** and **inc**, can be used to construct the theory of finite undirected graphs and in the framework it can look like this.

Structure-2. block __U

rus	Пусть **U** это какое-то конечное множество.
eng	Let U be some finite set.
yfl	declaration U fset(**Ide**) primary. –rus:U есть, задаваемое поэлементно, конечное подмножество счётного множества Ide.
flo	declaration U (). axiom Set(U). axiom U_1 ∀(≝e:U Ide(e)). –rus:позднее U будет задано поэлементно.

This frame [1] element has a unique identifier __U and the header "rus:основное множество"

Structure-3. block __inc

rus	Пусть **inc** это бинарное отношение на U. Будем называть его *отношением инцидентности*: "(a inc b)" читается "a инцидентно b".
eng	Let inc be a binary relation on U. We will call it an *incidence relation*: "(a inc b)" reads "a is incident to b".
yfl	declaration inc frel(U U) primary. –rus:inc есть бинарное отношение на U.
flo	declaration inc () . axiom inc_1 Rel(inc). axiom inc_2 ∀(≝p:inc U(1(p))&U(2(p))). –rus:inc предполагается заполнить парами как бинарное отношение на U.

Structure-4. components as global framework variables

In a framework, the elements of the structure (in this case U, inc) are considered global variables of the framework, i.e. they can be used in a frame element as known entities.

It is assumed that the value of global variables is set at the time of any calculation on them. Actually, specifying specific values is specifying a specific structure, the calculation on which will give a result that is interesting for the subject area. In mathematics, setting the values of global variables is setting a specific structure.

Axioms-1

Structure itself is a very general entity, we need a structure that satisfies certain requirements, i.e. having some properties. In theory, these mandatory properties are called axioms, and the structures that satisfy them are called models of axioms and the constructed theory.

For example, in the theory of undirected graphs, two axioms are needed and, accordingly, two blocks in the framework, having identifiers _INC01, _INC02.

Axioms-2. block _INC01

rus	inc удовлетворяет свойству INC01 если и только если члены встречающиеся в парах inc слева не встречаются справа.
eng	inc satisfies the INC01 property if and only if members occurring in inc pairs on the left do not occur on the right.
yfl	axiom INC01 ∀e,p:inc e(1)≠p(2).
flo	axiom INC01 ∀(≝e:inc ∀(≝p:inc 1(e)≠2(p))).

Axioms-3. block _INC02

rus	inc обладает свойством INC02 если и только если члены встречаются в парах inc слева только один или два раза.
eng	inc has the INC02 property if and only if the members occur only once or twice in inc pairs on the left.
yfl	axiom INC02 ∀e:inc (#y:inc e.1=y.1)≤2.
flo	axiom INC01 ∀(≝e:inc #(≝y:inc 1(e)=1(y))≤2)).

It is easy to notice that the specified properties concern only inc, i.e. can be tested on any binary relation.

Definitions of theory terms-1

The main content of the framework consists of definitions of terms, because terms are the way a person comprehends a subject area.

The number of theory terms can reach hundreds and in technologies even thousands and millions.

Formulating meaningful definitions is often a completely non-trivial process. Details can be found, for example, in [4].

Let's look at a few examples from the theory of undirected graphs. Each definition in the framework is a separate block, which is embedded into the texts of textbooks and, if necessary, articles.

Definitions of theory terms-1. block graphA

rus	Пусть g - совокупность элементов из U. g есть граф если и только если каждый элемент из g инцидентен только элементам из g.
eng	Let g be a collection of elements from U. g is a graph if and only if each element from g is incident only to elements from g.
yfl	declaration graph func(TV fset(U)) (g) ≝ ∀e:g (∀u:U (e inc u) → g(u)).
yfo	declaration graph (≝g:Fseq {Set(g)&∀(≝e:g U(e))} ∀(≝e:g (∀(≝u:U inc((e u)) → g(u))))). theorem graph_1 ∀(≝g:Fseq {Set(g)&∀(≝e:g U(e))} TV(graph(g))).

Definitions of theory terms-2. block edge

rus	Пусть x - элемент из U. x есть **ребро** если и только если x инцидентно какому-то элементу из U.
eng	Let x be an element of U. x is an edge if and only if x is incident to some element of U.
yfl	declaration edge func(TV U) (x) $\stackrel{\text{def}}{=}$ (∃y:U (x inc y)).
flo	declaration edge ($\stackrel{\text{def}}{=}$x:U ∃($\stackrel{\text{def}}{=}$y:U inc((x y)))). theorem edge_1 ∀($\stackrel{\text{def}}{=}$x:U TV(edge(x))).

Definitions of theory terms-3. block vertex

rus	пусть x есть элемент U. x есть **вершина** если и только если x не есть ребро.
eng	let x be an element of U. x is a vertex if and only if x is not an edge.
yfl	declaration vertex func(TV U) (x) ≝ not edge(x). –rus:синтсах: внешние скобки опущены
flo	declaration vertex (≝x:U (not edge(x))). theorem vertex_1 ∀(≝x:U TV(vertex(x))).

Definitions of theory terms-4. block d

Now we get into the terminology of undirected graphs.

rus	Пусть g - граф, v вершина, такая что v из g. степень вершины v в графе g есть количество простых рёбер графа g инцидентных v и количество петель графа g инцидентных v умноженное на два.
eng	Let g be a graph, v is a vertex such that v is in g. The degree of a vertex v in a graph g is the number of simple edges of the graph g that are incident with v and the number of loops of the graph g that are incident with v multiplied by two.
yfl	declaration d func(Nat graph vertex) (g v) $\stackrel{\text{def}}{=}$ {vertex(g)(v)} (#e:edge(g) ¬loop(e) & (e inc v)) + 2*(#e:edge(g) loop(e) & (e inc v)).

Theorem and hypothesis-1

The theorem first appears as an unproven hypothesis about the properties of the structure being studied. A hypothesis can be refuted, proven, or left as is.

The structure of the hypothesis and theorem block is the same as that of the axiom i.e. a property of structures.

For example, here is a theorem from the theory of undirected graphs with the identifier Th1_1 in its framework:

Theorem and hypothesis-2. block Th1_1

rus	Во всяком графе g сумма степеней вершин графа g равна удвоенному числу рёбер графа g.
eng	In any graph g, the sum of the degrees of the vertices of the graph g is equal to twice the number of edges of the graph g.
yfl	theorem Th1_1 ∀g:graph (Σv:vertex(g) d(g v)) = 2*(#x:g edge(g)(x)).

The agreement is that a theorem without proof is a hypothesis.

Proof-1

In the simplest case, a proof is a sequence of statements where the next statement has either already been proven earlier and is only included in this proof, or follows from the previous ones in the proof and then it is indicated which ones.

The last statement of the proof is the statement of the theorem that needs to be proven.

The theorem ID is in the proof header in the framework.

Proof-2. block Pr1_1__1

rus	1	каждое простое ребро даёт вклад 2 в сумму степеней вершин графа.	[simpleE]	"по определению"
rus	2	каждая петля даёт вклад 2 в степень своей вершины.	[d]	"по определению"
rus	3	каждое ребро графа даёт вклад равный двум в сумму степеней вершин графа.	[1 2]	"объединение"
rus	4	Во всяком графе g сумма степеней вершин графа g равна удвоенному числу рёбер графа g.	[3]	"суммирование"

eng	1	each simple edge contributes 2 to the sum of the degrees of the vertices of the graph.	[simpleE]	"a-priory"
eng	2	each loop contributes 2 to the degree of its vertex.	[d]	"a-priory"
eng	3	Each edge of the graph makes a contribution equal to two to the sum of the degrees of the vertices of the graph.	[1 2]	"union"
eng	4	In any graph g, the sum of the degrees of the vertices of the graph g is equal to twice the number of edges of the graph g.	[3]	"summation"

Proof-3

The section of each language, in addition to the language identifier, contains the <u>number of the sentence</u> in this section.

Moreover, now for each sentence the identifier of the frame element on which this sentence <u>is based</u> is indicated (usually this is a definition) or numbers of sentences of a given proof from which the current sentence follows. This column is called "<u>premises</u>".

The last column indicates <u>how</u> the current sentence is obtained from the sentences specified in the list of premises. This column is called "<u>method of inference</u>". The study of real methods of inference found in proofs is a separate important work, because these are not usually the rules of inference of formal logic.

The framework as a whole. maintenance, usage, development

The framework of a theory of, for example, undirected graphs:

- is maintained by the mathematical community and is available to everyone;
- contains verified, generally accepted definitions and all the hypotheses and theorems along with proofs;
- is used via embedding in textbooks whose content depends on the characteristics of the author and the audience.
- is used via embedding in articles. And new achievements will be checked and recorded in the framework.

- __inc rus:инцидентно
- __U rus:основное множество
- _INC01 rus:INC01 eng:INC01
- _INC02 rus:INC02 eng:INC02
- adjacent_e rus:смежно
- adjacent_v rus:смежна
- d rus:степень вершины в графе eng:degree
- edge rus:ребро eng:edge
- empty rus:пустой граф eng:???
- enp rus:концевая вершина ребра eng:endpoint of the edge
- graphA rus:граф eng:graph
- graphC rus:граф
- leaf rus:висячая вершина в графе
- loop rus:петля eng:loop
- null rus:нуль-граф граф eng:???
- order rus:порядок графа
- parallel rus:параллельно
- pendant rus:висячее ребро в графе
- Pr1_1__1 Th1_1

Future plans

In mathematics, the creation and maintenance of the framework of a particular theory is purely a matter of desire. And since they are well-known conservatives, we can expect that projects like HoTT or Wolfram Mathematica will like the idea.

The situation **in the natural sciences** is much more interesting: with the terms and definitions, implied structures, and used methods of constructing logical conclusions. Here we are supposed to take the science of genomics and see <u>in what form theoretical knowledge exists</u> in the formal ontology GENO [5].

Summary and analysis of inference methods. Here the transitions, leaps, and sometimes omissions of reasoning are studied.

Usage scenario

The framework of undirected graphs is the folder [1] in Google docs open to everyone for commenting.

The report, in the version stored in Google docs [2], is an example of using framework elements by embedding them in the current document.

The main type of texts describing the theory are textbooks in which, similarly to this document, elements of the framework will be embedded in.

The development of a theory occurs in articles when, for example, a new hypothesis is presented in the article, possibly immediately with a proof, and after checking it by the community, it, along with the proof, is added to the framework by the administrator.

Advantages

Maintaining one framework of theory for everyone should lead to a powerful concentration of knowledge and consistency of understanding, as well as facilitating familiarization with accumulated theoretical knowledge.

It is obvious that the formation of a framework for the theory presented in the textbook is a large collective effort.

Another advantage is the more structured storage of formalizations of theory elements, again in one place. It is from the framework that a set of formalizations will be used for one or another practical task.

AS2do

The difference between the theory framework and modern formal ontologies, for example, on OWL2, is the subject of a separate post.

Framework infrastructure, including access rights, etc. must be carefully thought out. At the moment, the capabilities of Google documents are used, which are sufficient for the prototype.

The application of the theory, including formalization, will be described in message No. 2. Where examples of the application of the theory in solving various tasks from [3] will be given. Extracting from the framework the elements necessary to solve a problem and constructing a solution to the problem is a separate important topic.

Every science contains one or more theories.

Theory is a systematic presentation of knowledge about a subject area.

Do you have a theory?

Framework it!

References

[1] framework

[2] Theory framework - knowledge hub. message #1 (-:PUBLIC:-)

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[4] S. Seppala, A. Ruttenberg, B. Smith, Guidelines for writing definitions in ontologies

[5] Genotype Ontology