Ontology-Based Data Access

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acknowledgements:

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Data Management: New Challenges

- Statoil (Norway)

  many databases, e.g., EPDS (Exploration and Production Data Store

  over 1500 tables

  historical exploration data (e.g., layers of rocks, porosity),

  production logs, maps, etc.

  business information such as license areas and companies

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- **Siemens Energy** (Germany)

  power generation facilities (gas and steam turbines)

  50 service centres linked to a common database

  each turbine

  2000 sensors

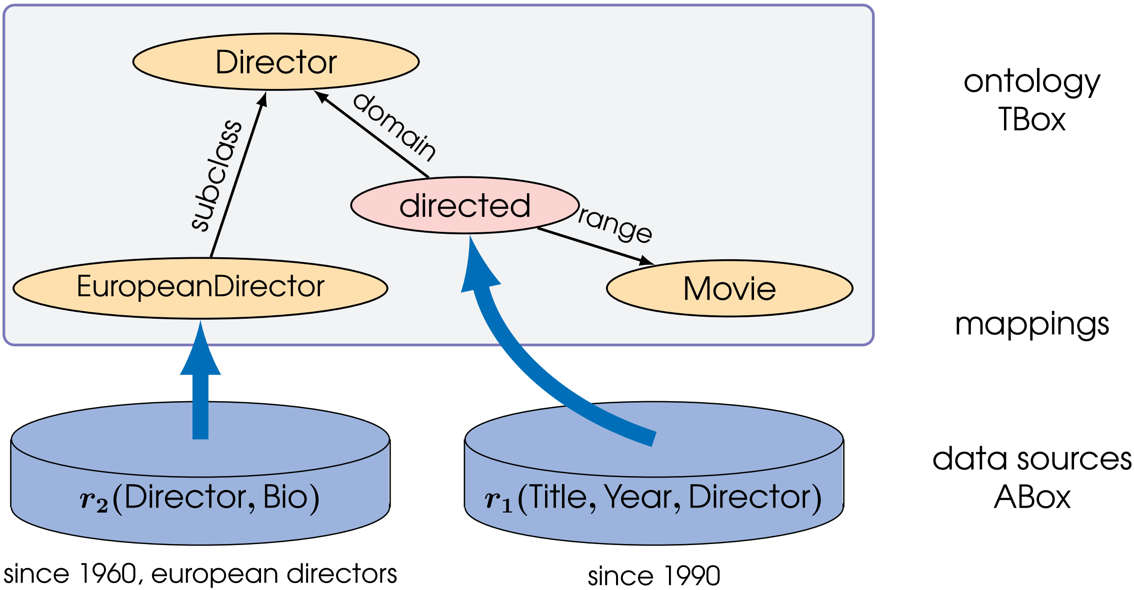
  150 tables

  30 GB of data is generated daily (hundreds of terabytes in total)

Ontology-Based Data Access

Aim: to achieve **logical transparency** in accessing data

- hide from the user where and how data is stored
- present only a **conceptual view** of the data
- **query** the data sources through the **conceptual model** using **RDBMSs**



Issues in OBDA

- what is the right ontology language?
  - there is a wide spectrum of languages that differ in
    expressive power and **complexity** of inference
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- how do we connect ontologies to data sources?
  - multiple data sources and ontologies

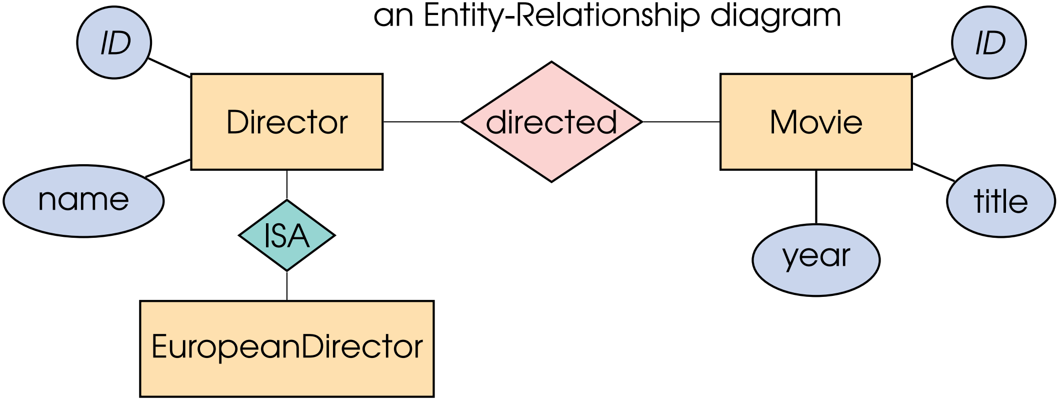
- available tools?
  - sound and complete reasoning
  - practical scalability

Part 1

Databases and Logic

Databases: Specifying Schema

an Entity-Relationship diagram



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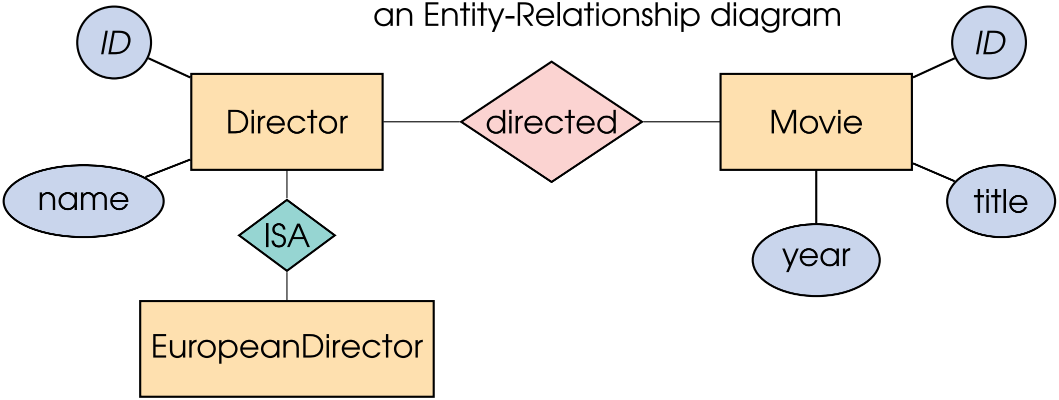

integrity constraints or dependencies (in the language of FO):

$\forall d\ (\exists m\ \text{directed}(d, m) \to \exists n\ \text{Director}(d, n))$ (foreign keys, inclusion or
$\forall m\ (\exists d\ \text{directed}(d, m) \to \exists ty\ \text{Movie}(m, t, y))$ tuple-generating dependencies,
$\forall dn\ (\text{EuropeanDirector}(d, n) \to \text{Director}(d, n))$ **TGDs**)

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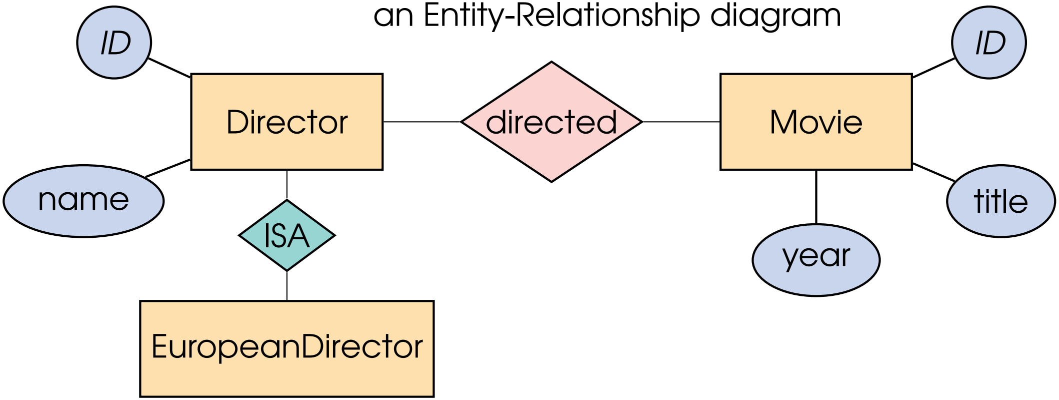

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(foreign keys, inclusion or tuple-generating dependencies, **TGDs**)

$\forall d n_1 n_2\,(\text{Director}(d,n_1) \wedge \text{Director}(d,n_2) \to (n_1 = n_2))$
$\forall m t_1 t_2 y_1 y_2\,(\text{Movie}(m,t_1,y_1) \wedge \text{Movie}(m,t_2,y_2) \to (t_1 = t_2))$

(keys, functional or equality-generating dependencies, **EGDs**)

Databases: Data and the Closed World Assumption

data is **completely** specified (**closed world assumption**) and is typically **large**

what is specified is true, everything else is false

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data:

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$$\text{Movie} = \{\ (\mathbf{10}, \text{“DC”}),\ \ (\mathbf{11}, \text{“TS”})\ \}$$

$$\text{directed} = \{\ (\mathbf{0}, \mathbf{10}),\ \ (\mathbf{2}, \mathbf{11})\ \}$$

query: $\boldsymbol{q}(\boldsymbol{n}) = \exists \boldsymbol{d}\, \text{Director}(\boldsymbol{d}, \boldsymbol{n})$

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answer: $\{\ \text{"peter"},\ \ \text{"quentin"},\ \ \text{"danny"}\ \}$

NB: not having $(\mathbf{2}, \text{"danny"})$ in Director would violate the integrity constraint

$$\forall \boldsymbol{dn}\ (\text{EuropeanDirector}(\boldsymbol{d}, \boldsymbol{n}) \rightarrow \text{Director}(\boldsymbol{d}, \boldsymbol{n}))$$

Databases: Query Languages

SQL $\approx$ domain-independent **FO queries**:

database predicates + logical connectives $\lor$, $\land$, $\neg$ + quantifiers $\forall$, $\exists$

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$\vec{a}$ is an **answer** to $q(\vec{x})$ iff $\mathcal{I}_{\mathcal{D}} \models q(\vec{a})$

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Select-Project-Join (SPJ) = **conjunctive queries** (CQs):

database predicates + $\land$ + $\exists$

database engines are optimised for CQs

Example: SELECT M.title, D.name
FROM Movie M, Directed MD, Director D
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Datalog notation:

$$\underbrace{q(\vec{x})}_{\text{head}} \leftarrow \underbrace{P_1(\vec{z}_1), \dots, P_k(\vec{z}_k)}_{\text{body}}$$

where each $\vec{z}_i$ is a vector, which may contain **answer variables** $\vec{x}$ and existentially quantified variables $\vec{y}$ (implicit)

Example: $q(t, n) \leftarrow$ Movie$(m, t, 1982)$, directed(m, d), director(d, n)

Why do Databases Work?

query answering problem (as a recognition problem):

given a finite data $\mathcal{D}$, a query $q(\vec{x})$ and a tuple $\vec{a}$,
decide whether $\mathcal{I}_{\mathcal{D}} \models q(\vec{a})$

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what is the complexity of CQ answering?

naive algorithm:

guess values for all existential variables and then
evaluate the query in polynomial time — in **NP**

can it be done better?

Why Do Databases Work? (2)

no, by reduction of the graph 3-colourability problem, which is **NP-complete**:

'given an undirected graph $G = (V, E)$,
decide whether it possible to colour it (using r, g, b)
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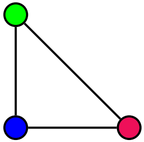

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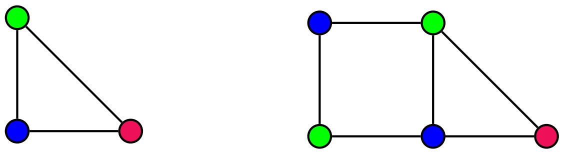

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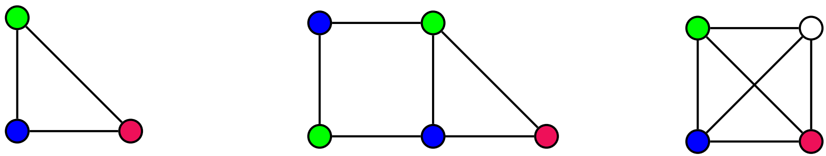

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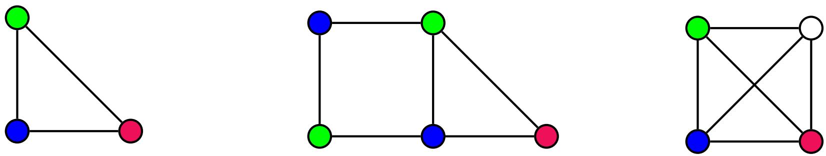
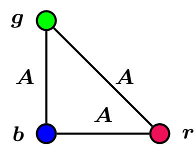


$$\mathcal{D} = \{A(r,g), A(g,b), A(b,r), A(g,r), A(r,b), A(b,g)\}$$

$$q_G = \exists v_1, \dots, v_n \bigwedge_{(v_i,v_j)\in E} A(v_i, v_j)$$



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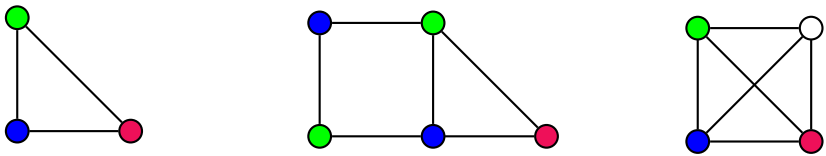

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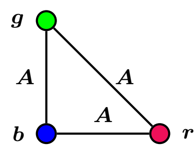

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in fact, the query answering algorithm runs in $O(|\mathcal{D}|^{|q|})$ **data is large, query is short**

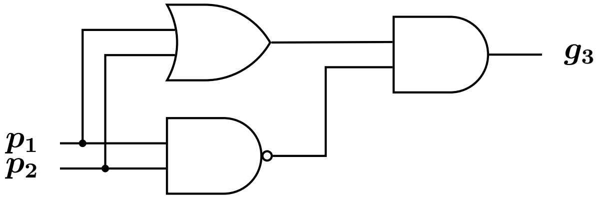
data complexity: only data $\mathcal{D}$ are counted as **input** (q is constant)

(Vardi, 1982): query answering is in AC^0 for data complexity

Circuits and AC^0

a **circuit** is an acyclic graph of AND-, OR- and NOT-gates

(with n inputs and a single output, **sink**)



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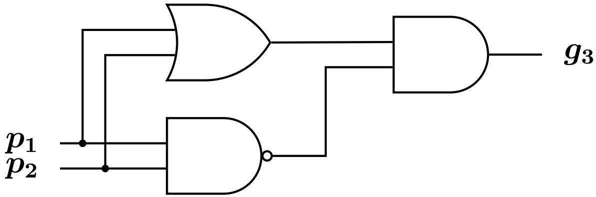

database instances $\mathcal{D}$ can be encoded on inputs

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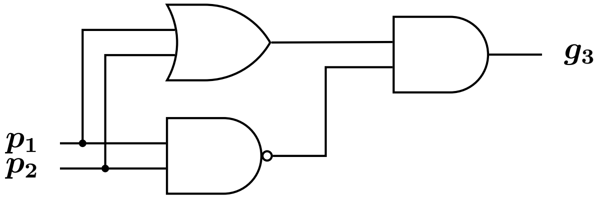

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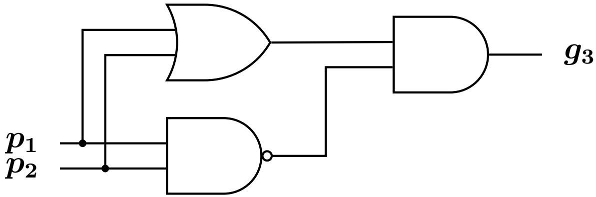

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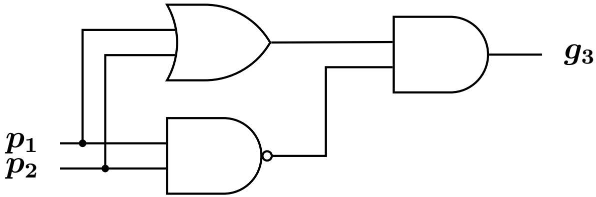

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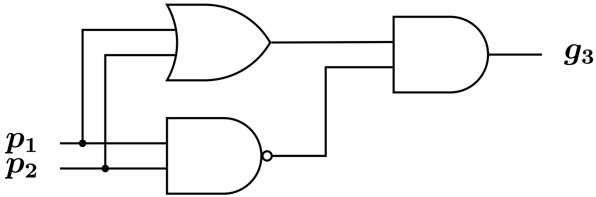

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NB: AC^0 is a **proper** subclass of LogSpace $\subseteq$ P (Parity does not belong to AC^0)

given a word w, decide whether its length is even

Part 2

Basics of Ontology Languages

DLs and OWL: Syntax

concepts (classes, sets of elements)

$$C \quad ::= \quad \underbrace{A_i}_{\text{concept name}} \quad | \quad \underbrace{\top}_{\text{owl:Thing}} \quad | \quad \underbrace{\bot}_{\text{owl:Nothing}} \quad |$$

$$\underbrace{\neg C}_{\text{ObjectComplementOf}(C)} \quad | \quad \underbrace{C_1 \sqcap C_2}_{\text{ObjectIntersectionOf}(C_1,C_2)} \quad | \quad \underbrace{C_1 \sqcup C_2}_{\text{ObjectUnionOf}(C_1,C_2)} \quad |$$

$$\underbrace{\exists R.C}_{\text{ObjectSomeValuesFrom}(R,C)} \quad | \quad \underbrace{\forall R.C}_{\text{ObjectAllValuesFrom}(R,C)}$$

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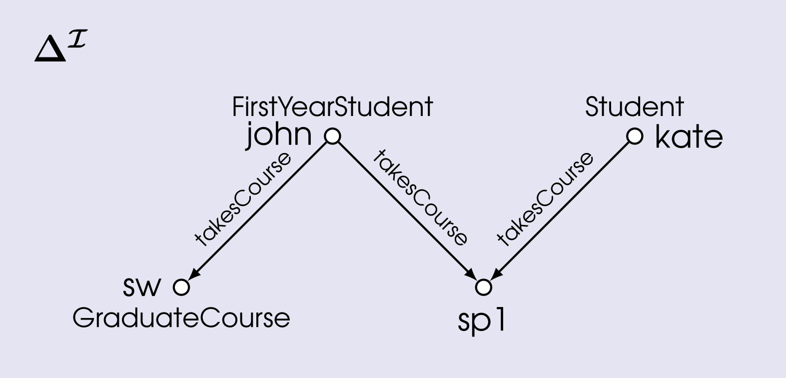
ABox $\mathcal{A}$ $\quad C(a)$ and $R(a, b)$

knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ (ontology)

DL Semantics

interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{\text{domain}}, \cdot^{\mathcal{I}})$

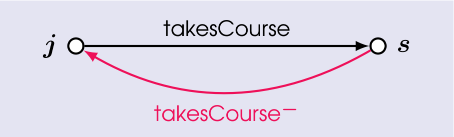


$\cdot^{\mathcal{I}}$ (interpretation function)

individuals a_i	$\rightarrow$	elements $a_i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
concept names A_i	$\rightarrow$	subsets $A_i^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
role names P_i	$\rightarrow$	binary relations $P_i^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

DL Semantics (2)

$$(P^{-})^{\mathcal{I}} \ = \ \{(v, u) \mid (u, v) \in P^{\mathcal{I}}\}$$



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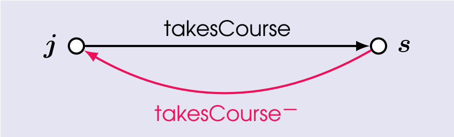

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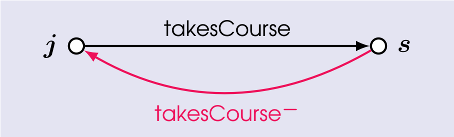

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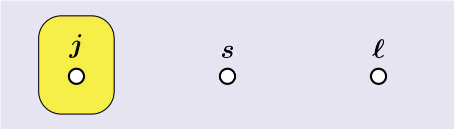
$$(\neg C)^\mathcal{I} \;=\; \Delta^\mathcal{I} \setminus C^\mathcal{I}$$



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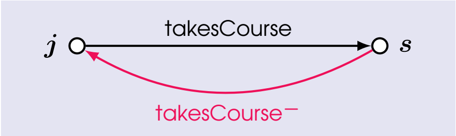
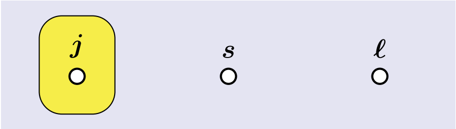


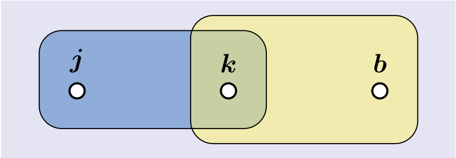
$$\top^\mathcal{I} = \Delta^\mathcal{I} \quad \text{and} \quad \bot^\mathcal{I} = \emptyset$$

$$(\neg C)^\mathcal{I} = \Delta^\mathcal{I} \setminus C^\mathcal{I}$$



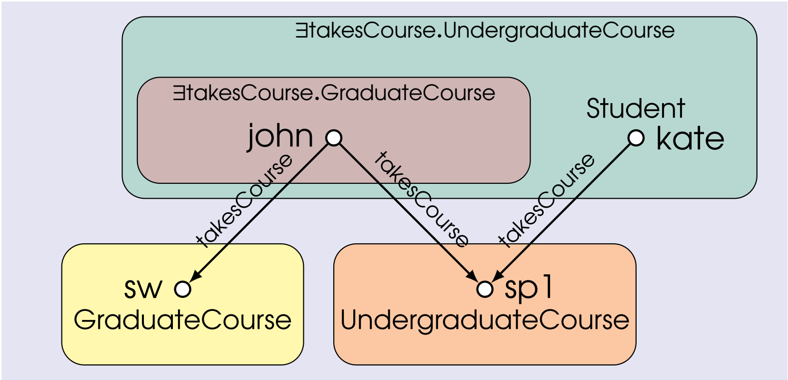
$$(C_1 \sqcap C_2)^\mathcal{I} = C_1^\mathcal{I} \cap C_2^\mathcal{I}$$

$$(C_1 \sqcup C_2)^\mathcal{I} = C_1^\mathcal{I} \cup C_2^\mathcal{I}$$



DL Semantics (3)

$$(\exists R.C)^{\mathcal{I}} = \{ u \mid \text{ there is } v \in C^{\mathcal{I}} \text{ such that } (u, v) \in R^{\mathcal{I}} \}$$



$\Diamond_R C$ or $\exists y\,(R(x,y) \wedge C(y))$

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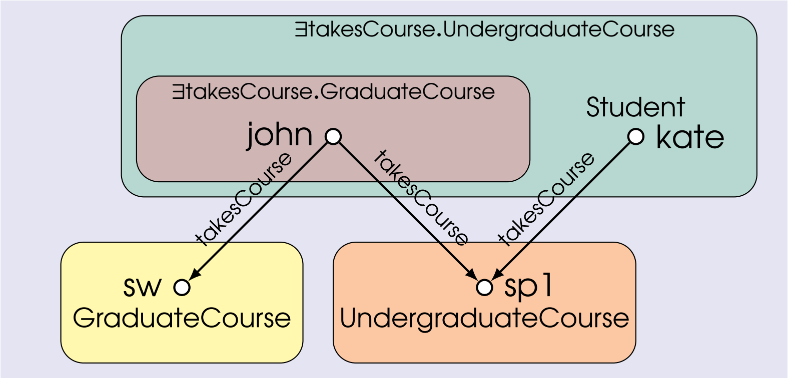

$\Diamond_R C$ or $\exists y\,(R(x, y) \land C(y))$

$$(\forall R.C)^{\mathcal{I}} = \{\, u \mid v \in C^{\mathcal{I}}, \text{ for all } v \text{ with } (u, v) \in R^{\mathcal{I}} \,\}$$

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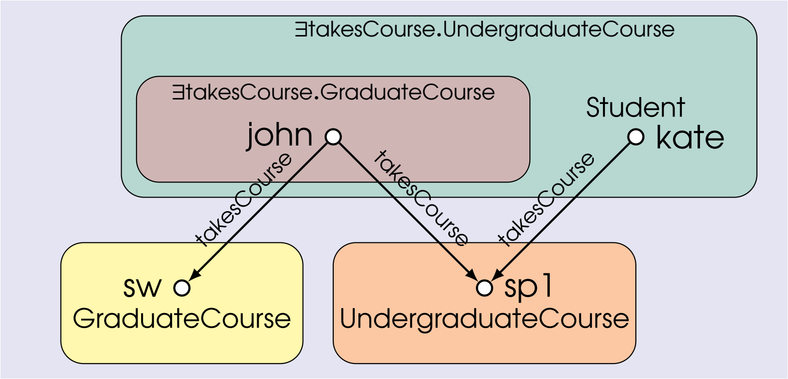

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NB. "for all" is true when there are no v with $(u,v) \in R^{\mathcal{I}}$

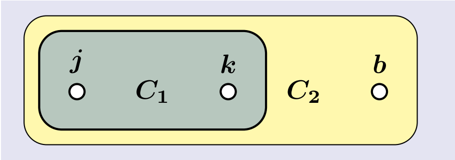
e.g., sp1 $\in$ ($\forall$takesCourse.UndergraduateCourse)$^{\mathcal{I}}$

sp1 $\in$ ($\forall$takesCourse.$\bot$)$^{\mathcal{I}}$

$\Box_R C$ or $\forall y\,(R(x,y) \to C(y))$

DL Semantics (4)

$$\mathcal{I} \models C_1 \sqsubseteq C_2 \iff C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$$



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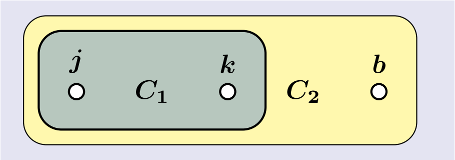

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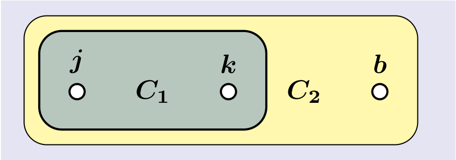
DL Semantics (4)



$$\mathcal{I} \models C_1 \sqsubseteq C_2 \iff C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$$

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$$\mathcal{I} \models C(a) \iff a^{\mathcal{I}} \in C^{\mathcal{I}}$$

$$\mathcal{I} \models R(a, b) \iff (a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$$

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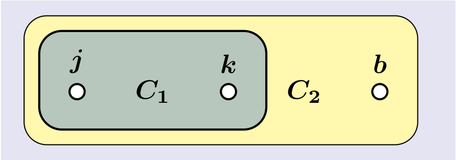

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$\mathcal{I}$ is a **model** of $(\mathcal{T}, \mathcal{A})$ if $\mathcal{I} \models \alpha$, for all inclusions α in $\mathcal{T}$ and assertions α in $\mathcal{A}$

Open World Assumption

$$\mathcal{T} = \{ \text{GraduateStudent} \sqsubseteq \text{Student}$$
$$\text{GraduateStudent} \sqsubseteq \exists \text{takesCourse}.\text{GraduateCourse} \}$$
$$\mathcal{A} = \{ \text{GraduateStudent(john)} \}$$

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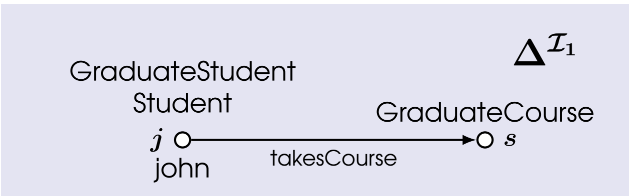

$\text{john}^{\mathcal{I}_1} = j$
$\text{GraduateStudent}^{\mathcal{I}_1} = \{j\}$
$\text{Student}^{\mathcal{I}_1} = \{j\}$
$\text{GraduateCourse}^{\mathcal{I}_1} = \{s\}$
$\text{takesCourse}^{\mathcal{I}_1} = \{(j, s)\}$

is a **model** of $(\mathcal{T}, \mathcal{A})$

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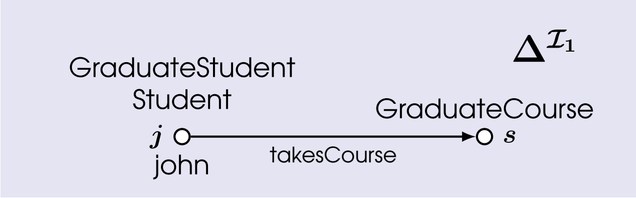

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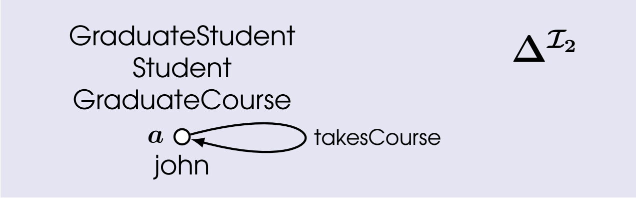

$\text{john}^{\mathcal{I}_2} = a$
$\text{GraduateStudent}^{\mathcal{I}_2} = \{a\}$
$\text{Student}^{\mathcal{I}_2} = \{a\}$
$\text{GraduateCourse}^{\mathcal{I}_2} = \{a\}$
$\text{takesCourse}^{\mathcal{I}_2} = \{(a, a)\}$
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Open World Assumption

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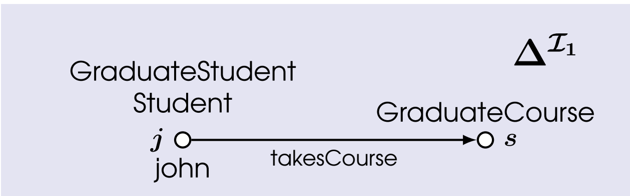

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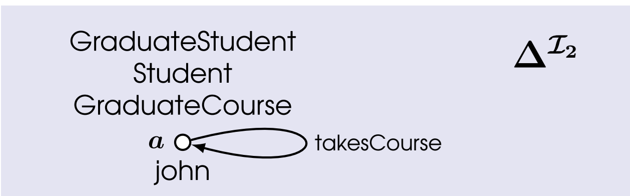

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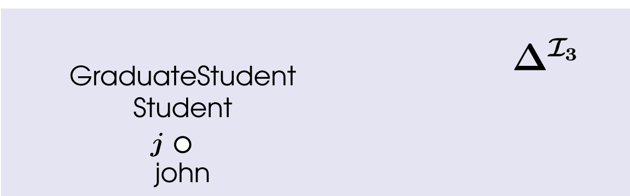

$\text{john}^{\mathcal{I}_3} = j$
$\text{GraduateStudent}^{\mathcal{I}_3} = \{j\}$
$\text{Student}^{\mathcal{I}_3} = \{j\}$
$\text{GraduateCourse}^{\mathcal{I}_3} = \emptyset$
$\text{takesCourse}^{\mathcal{I}_3} = \emptyset$
is **not a model** of $(\mathcal{T}, \mathcal{A})$

Reasoning: Consistency

a knowledge base $\mathcal{K}$ is **satisfiable** (or **consistent**)

if there exists at least one model of $\mathcal{K}$

(in other words, $\mathcal{K}$ implies **no contradictions**)

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Example

$\mathcal{T}$:

$\text{UndergraduateStudent} \sqsubseteq \forall \text{takesCourse.UndergraduateCourse}$

$\text{UndergraduateCourse} \sqcap \text{GraduateCourse} \sqsubseteq \bot$

$\mathcal{A}$:

UndergraduateStudent(john)

takesCourse(john, sw)

GraduateCourse(sw)

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$\mathcal{A}$:

$$\mathsf{UndergraduateStudent}(\mathsf{john})$$

$$\mathsf{takesCourse}(\mathsf{john}, \mathsf{sw})$$

$$\mathsf{GraduateCourse}(\mathsf{sw})$$

$(\mathcal{T}, \mathcal{A})$ is **inconsistent**:

John (as an undergraduate student) can take only undergraduate courses.
We know, however, that he takes a graduate course,
which cannot be an undergraduate one.

Reasoning: Entailment

$C_1 \sqsubseteq C_2$ is **entailed by** $\mathcal{K}$ $\qquad$ $\mathcal{K} \models C_1 \sqsubseteq C_2$

if $\mathcal{I} \models C_1 \sqsubseteq C_2$ for all models $\mathcal{I}$ of $\mathcal{K}$

(entailment for role inclusions and concept and role assertions is defined similarly)

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$\mathcal{T}$: $\forall$takesCourse.UndergraduateCourse $\sqsubseteq$ UndergraduateStudent

FirstYearStudent $\sqsubseteq$ $\exists$takesCourse.UndergraduateCourse.

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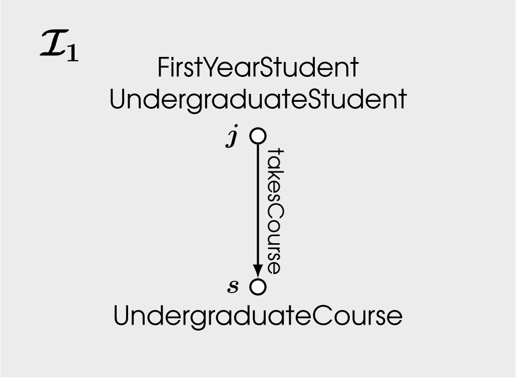

$\mathcal{I}_1 \models \mathcal{T}$

$\mathcal{I}_1 \models$ FirstYearStudent $\sqsubseteq$ UndergraduateStudent

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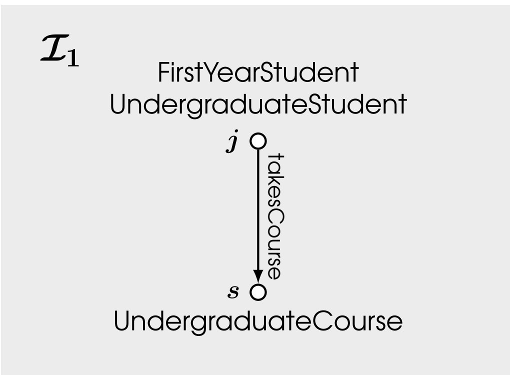

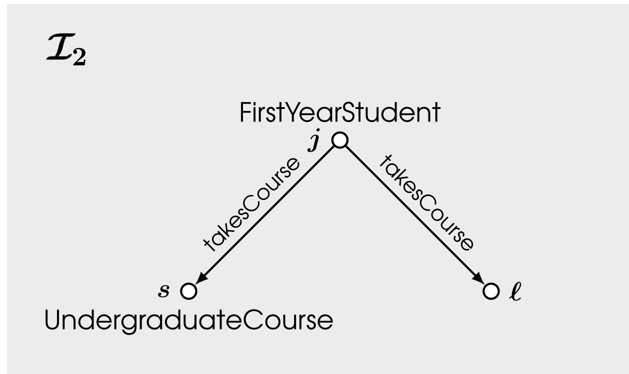

$\mathcal{I}_1 \models \mathcal{T}$

$\mathcal{I}_1 \models$ FirstYearStudent $\sqsubseteq$ UndergraduateStudent

$\mathcal{I}_2 \models \mathcal{T}$

$\mathcal{I}_2 \models$ FirstYearStudent $\not\sqsubseteq$ UndergraduateStudent

Certain Answers to CQs

$q(\vec{x}) = \exists \vec{y}\, \varphi(\vec{x}, \vec{y})$ is a CQ with $\vec{x} = (x_1, \dots, x_n)$

$\vec{a} = (a_1, \dots, a_n)$ is a tuple of individual names from $\mathcal{A}$

$q(\vec{a})$ is the result of replacing each x_i in $\exists \vec{y}\, \varphi(\vec{x}, \vec{y})$ with a_i

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$\vec{a}$ is a **certain answer** to $q(\vec{x})$ over $\mathcal{T}, \mathcal{A}$ $\quad$ $(\mathcal{T}, \mathcal{A}) \models q(\vec{a})$

if, <u>for any model</u> $\mathcal{I}$ of $(\mathcal{T}, \mathcal{A})$, the sentence $q(\vec{a})$ is true in $\mathcal{I}$

$\mathcal{I} \models q(\vec{a})$

Andrea's Example (Schaerf, 1993)

$\mathcal{T}$: $\top \sqsubseteq \mathsf{Male} \sqcup \mathsf{Female}$, $\mathsf{Male} \sqcap \mathsf{Female} \sqsubseteq \bot$

$\mathcal{A}$: friend(john, susan), friend(john, andrea), Female(susan)
loves(susan, andrea), loves(andrea, bill), Male(bill)

$$q = \exists y, z \left(\mathsf{friend}(\mathsf{john}, y) \wedge \mathsf{Female}(y) \wedge \mathsf{loves}(y, z) \wedge \mathsf{Male}(z)\right)$$

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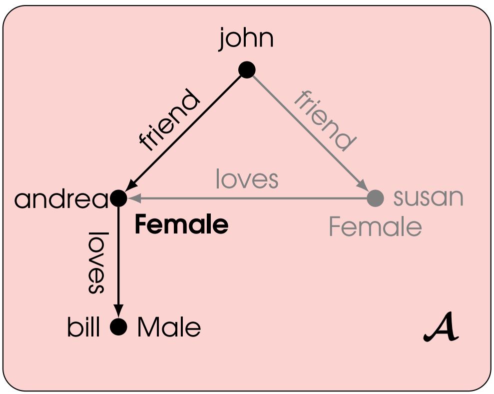

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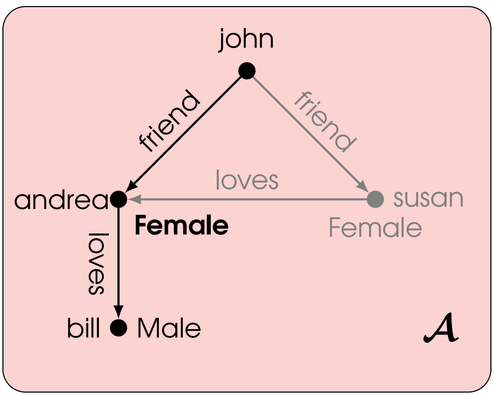

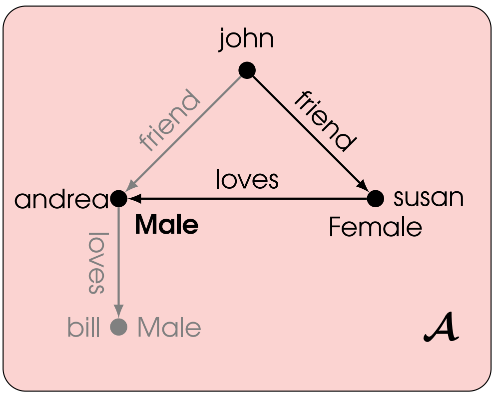

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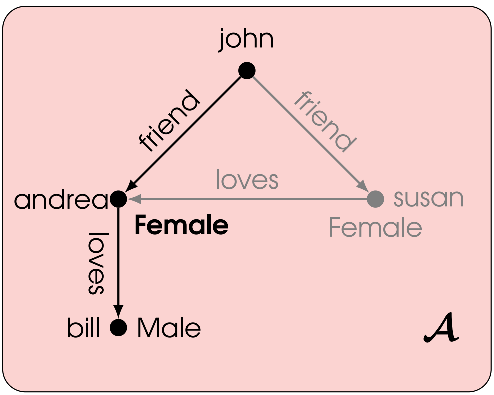

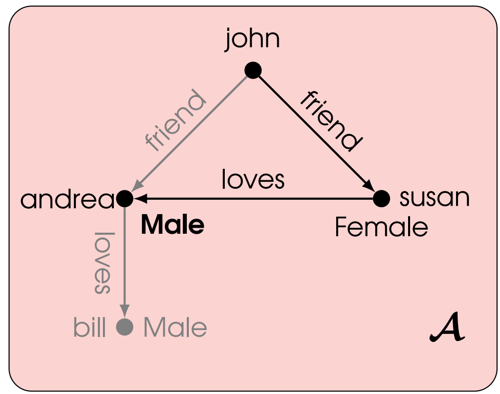

NB: the same as checking whether john is an instance of $\exists$friend.(Female $\sqcap$ $\exists$loves.Male)

DL Zoo

$\mathcal{ALCHI}$

$\mathcal{AL}$ – attributive language

$\mathcal{C}$ – complement $\neg C$ ($\mathcal{ALC}$ is multi-modal $\mathbf{K}_m$)

$\mathcal{I}$ – role inverses P^-

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$\mathcal{S}$ – $\mathcal{ALC}$ + transitive roles

- $\mathcal{N}$ – unqualified number restrictions $\geq q\, R.\top$
- $\mathcal{O}$ – nominals $\{a\}$

$\mathcal{SHOIN} \approx$ OWL 1.0

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$\mathcal{SHIF} \approx$ OWL Lite

$\mathcal{R}$ – role chains and $\exists R.$Self

$\mathcal{SROIQ} \approx$ OWL 2

Complexity of Reasoning

The satisfiability problem is **ExpTime**-complete for $\mathcal{ALCHI}$ KBs
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DL Complexity Navigator: `www.cs.man.ac.uk/~ezolin/dl`

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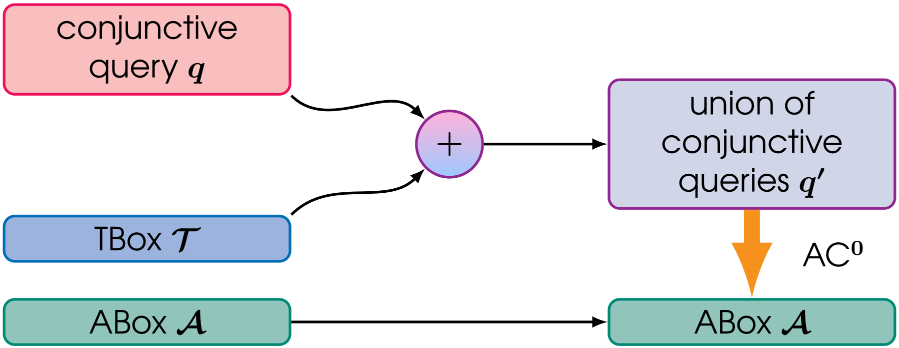
practical reasoners for OWL 2 DL: FaCT++, HermiT, Pellet

Part 3

Conjunctive Query Rewriting

Query Rewriting Approach

(Calvanese et al. 2008): use off-the-shelf RDBMS



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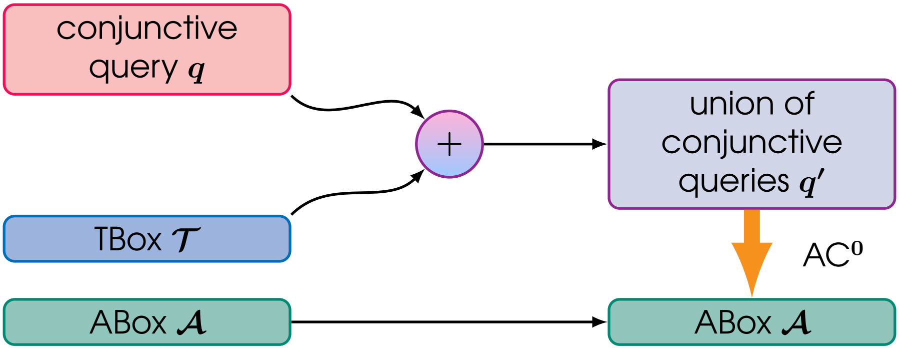

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for all $\mathcal{A}$ and $\vec{a}$, $\quad \mathcal{T}, \mathcal{A} \models q(\vec{a}) \quad$ iff $\quad \mathcal{A} \models q'(\vec{a})$

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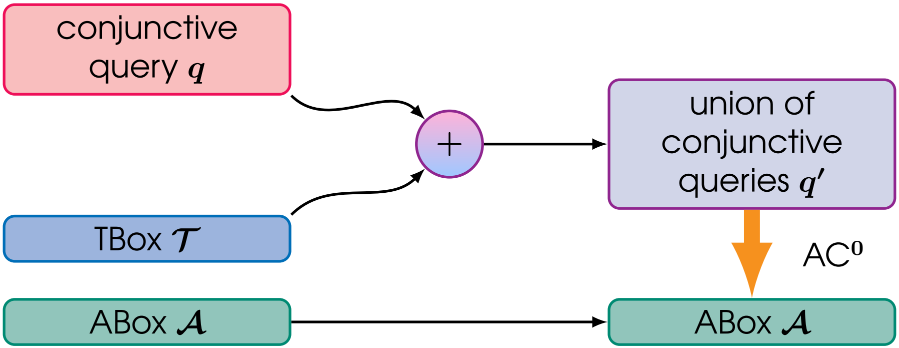

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FO-rewritability: only possible in DL with query answering in **FO** (=AC^0) for data complexity:

OWL 2 QL

W3C Standard OWL 2 QL

OWL 2 QL is a profile of **OWL 2** designed with the aim of OBDA

(and based on the *DL-Lite* family of DLs)

roles	R	$::=$	P_i	$\mid$	P_i^-		
basic concepts	B	$::=$	$\bot$	$\mid$	A_i	$\mid$	$\exists R$
concepts	C	$::=$	B	$\mid$	$\exists R.B$		

($\exists R$ is an abbreviation for $\exists R.\top$)

a TBox $\mathcal{T}$ is a finite set of axioms of the form

$$B \sqsubseteq C, \qquad R_1 \sqsubseteq R_2, \qquad B_1 \sqcap B_2 \sqsubseteq \bot, \qquad R_1 \sqcap R_2 \sqsubseteq \bot$$

(plus reflexivity/irreflexivity assertions for roles)

an ABox $\mathcal{A}$ s a finite set of atoms the form $A_k(a_i)$ and $P_k(a_i, a_j)$

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Can We Use $\exists R.A \sqsubseteq B$ in QL?

reachability problem for directed graphs is **NLogSpace-complete**:

'given a directed graph $G = (V, E)$ and $s, t \in V$, decide whether there is a directed path from s to t in G'

ABox: $\mathcal{A}_{G,t} = \{ \text{edge}(v_1, v_2) \mid (v_1, v_2) \in E \} \cup \{\text{ReachableFromTarget}(t)\}$

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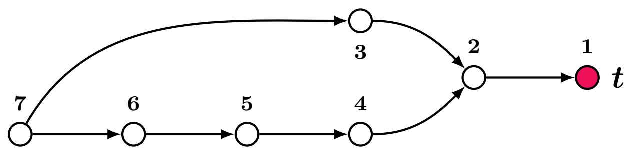

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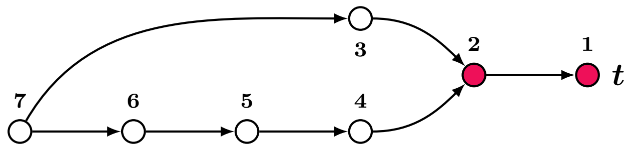

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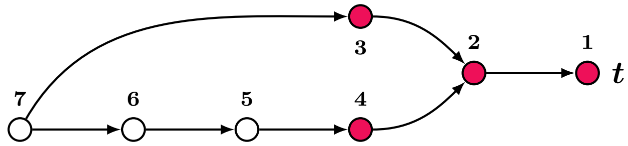

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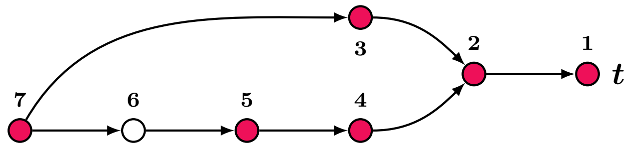

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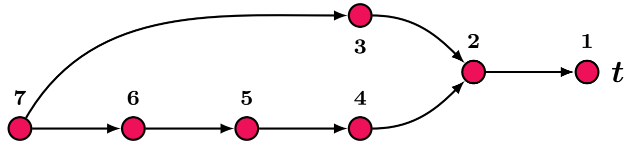

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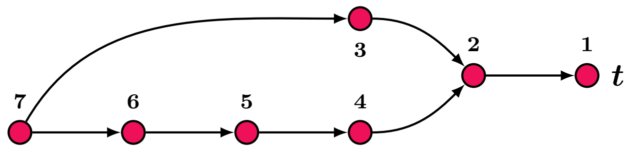

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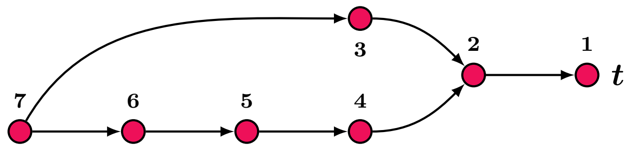

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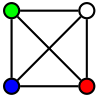
$\mathcal{T}$ and q do not depend on G, s, t

→ '$(\mathcal{T}, \mathcal{A}_{G,t}) \models q$?' is **NLogSpace**-hard for data complexity

→ q and $\mathcal{T}$ are not FO-rewritable

Can We Use $A \sqsubseteq B \sqcup C$ in QL?



graph 3-colouring problem is **NP-complete**:

'given a graph $G = (V, E)$, decide whether its vertices can be painted in one of three colours so that no adjacent vertices have the same colour'

represent G as the ABox $\mathcal{A}_G = \{ R(v_1, v_2) \mid \{v_1, v_2\} \in E \}$

3-colouring is encoded by the TBox $\mathcal{T}$ with the axioms

$$\top \sqsubseteq C_1 \sqcup C_2 \sqcup C_3, \qquad C_i \sqcap C_j \sqsubseteq \bot, \qquad C_i \sqcap \exists R.C_i \sqsubseteq B, \quad 1 \leq i \leq 3$$

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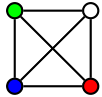

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consider CQ $q \leftarrow B(y)$

$(\mathcal{T}, \mathcal{A}_G) \not\models q$ iff G is 3-colourable

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OWL 2 QL as TGDs

(aka Datalog$^{\pm}$ aka existential rules)

concept inclusion		tuple-generating dependency
PhDStudent $\sqsubseteq$ Student	$\approx$	$\forall x\,(\text{PhDStudent}(x) \rightarrow \text{Student}(x))$
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linear TGDs: φ and ψ are atoms (all *OWL 2 QL* axioms are linear)

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NB: these TGDs are used with the **open world assumption** (for enriching data)

TGDs in **DBs** are used with the **closed world assumption** (integrity constraints)

Practical Query Answering in OWL 2 QL

systems

- QuOnto (Rome, 2005)
- REQUIEM (Oxford, 2009) / Stardog (Washington, DC, 2011)
- Presto (Rome, 2010)
- IQAROS (Athens, 2011)
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- Nyaya (Milan-Oxford, 2010) for TGDs
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not so smoothly: the size of implemented rewritings q' is $O\big((|q| \cdot |\mathcal{T}|)^{|q|}\big)$

(can't say 'query is small or fixed' any longer)

Does a Rewriting Have to be Exponential?

TBox	mother $\sqsubseteq$ parent and father $\sqsubseteq$ parent
query	grandparent$(x, z) \leftarrow$ parent$(x, y) \wedge$ parent(y, z)

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NDL-rewriting (non-recursive Datalog ≈ SQL with views)

$\exists \ \lor \ \land$ + structure sharing

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FO-rewriting (first-order queries ≈ SQL) $\exists \forall \ \lor \ \land \neg$

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a TBox $\mathcal{T}$ is **flat** if it does not contain **generating axioms** $B' \sqsubseteq \exists R.B$

≈ RDF Schema

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$q(\vec{x})$ and a flat $\mathcal{T}$ → $q_{\text{ext}}(\vec{x})$ by replacing

$$A(u) \longrightarrow \bigvee_{\mathcal{T} \models A' \sqsubseteq A} A'(u) \lor \bigvee_{\mathcal{T} \models \exists R \sqsubseteq A} \exists v\, R(u,v)$$

$$P(u,v) \longrightarrow \bigvee_{\mathcal{T} \models R \sqsubseteq P} R(u,v)$$

for any CQ $q(\vec{x})$ and any flat OWL 2 QL TBox $\mathcal{T}$,
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easy in theory, not so in practice

Who Works with Professors?

TBox:

$$\text{worksOn}^{-} \sqsubseteq \text{involves}$$
$$\text{isManagedBy} \sqsubseteq \text{involves}$$

in English: find those who work with professors

query: $q(x) \leftarrow \text{worksOn}(x, y) \wedge \text{involves}(y, z) \wedge \text{Professor}(z)$

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Rewriting over H-complete ABoxes

an ABox $\mathcal{A}$ is **H-complete with respect to $\mathcal{T}$** if

- $A(a) \in \mathcal{A}$ whenever $A'(a) \in \mathcal{A}$ and $\mathcal{T} \models A' \sqsubseteq A$
- $A(a) \in \mathcal{A}$ whenever $R(a, b) \in \mathcal{A}$ and $\mathcal{T} \models \exists R \sqsubseteq A$
- $P(a, b) \in \mathcal{A}$ whenever $R(a, b) \in \mathcal{A}$ and $\mathcal{T} \models R \sqsubseteq P$

Rewriting over H-complete ABoxes

an ABox $\mathcal{A}$ is **H-complete with respect to $\mathcal{T}$** if

- $A(a) \in \mathcal{A}$ whenever $A'(a) \in \mathcal{A}$ and $\mathcal{T} \models A' \sqsubseteq A$
- $A(a) \in \mathcal{A}$ whenever $R(a, b) \in \mathcal{A}$ and $\mathcal{T} \models \exists R \sqsubseteq A$
- $P(a, b) \in \mathcal{A}$ whenever $R(a, b) \in \mathcal{A}$ and $\mathcal{T} \models R \sqsubseteq P$

an FO-query $q'(\vec{x})$ is an **FO-rewriting of $q(\vec{x})$ and $\mathcal{T}$ over H-complete ABoxes** if,

for any H-complete (w.r.t. $\mathcal{T}$) ABox $\mathcal{A}$ and any $\vec{a}$,

$(\mathcal{T}, \mathcal{A}) \models q(\vec{a})$ iff $\mathcal{A} \models q'(\vec{a})$

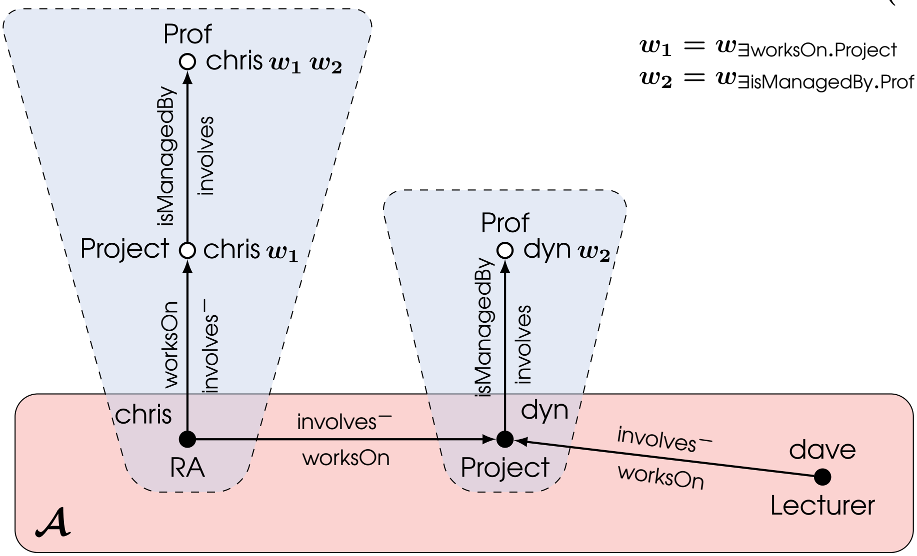
(thus we ignore the axioms considered in the flat rewriting)

Case 2: Who Works with Professors (2)?

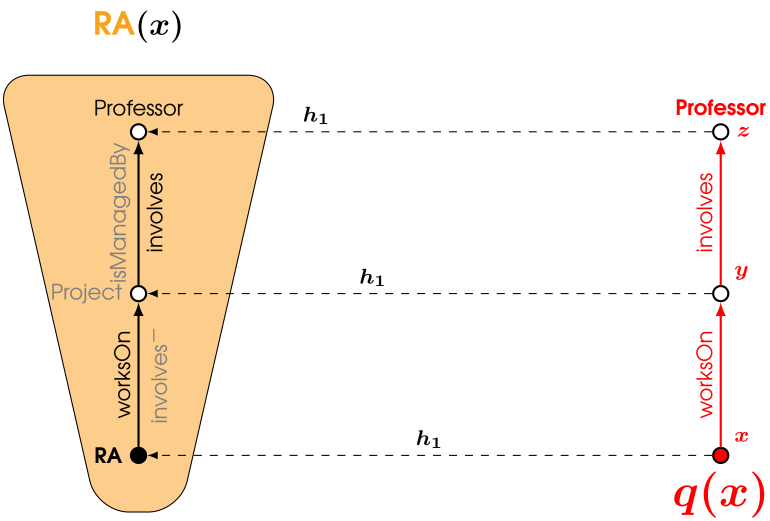
$\mathcal{T}$: RA $\sqsubseteq$ ∃worksOn.Project   worksOn$^-$ $\sqsubseteq$ involves
Project $\sqsubseteq$ ∃isManagedBy.Prof   isManagedBy $\sqsubseteq$ involves

$\mathcal{A}$: RA(chris), worksOn(chris, dyn), Project(dyn), Lecturer(dave),
worksOn(dave, dyn)

$w_1 = w_{\exists\text{worksOn.Project}}$
$w_2 = w_{\exists\text{isManagedBy.Prof}}$



Case 2: Rewriting the Labelled Nulls



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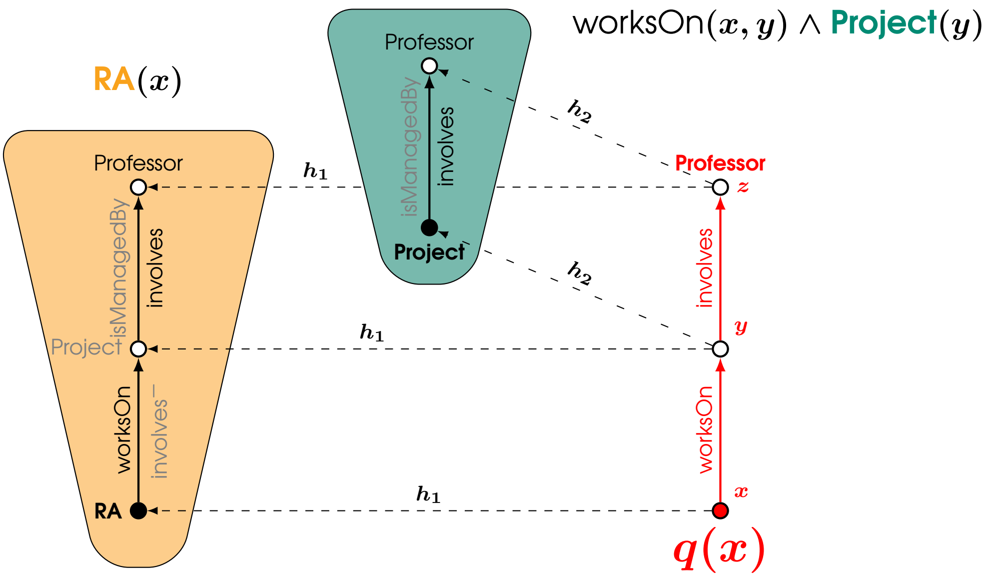

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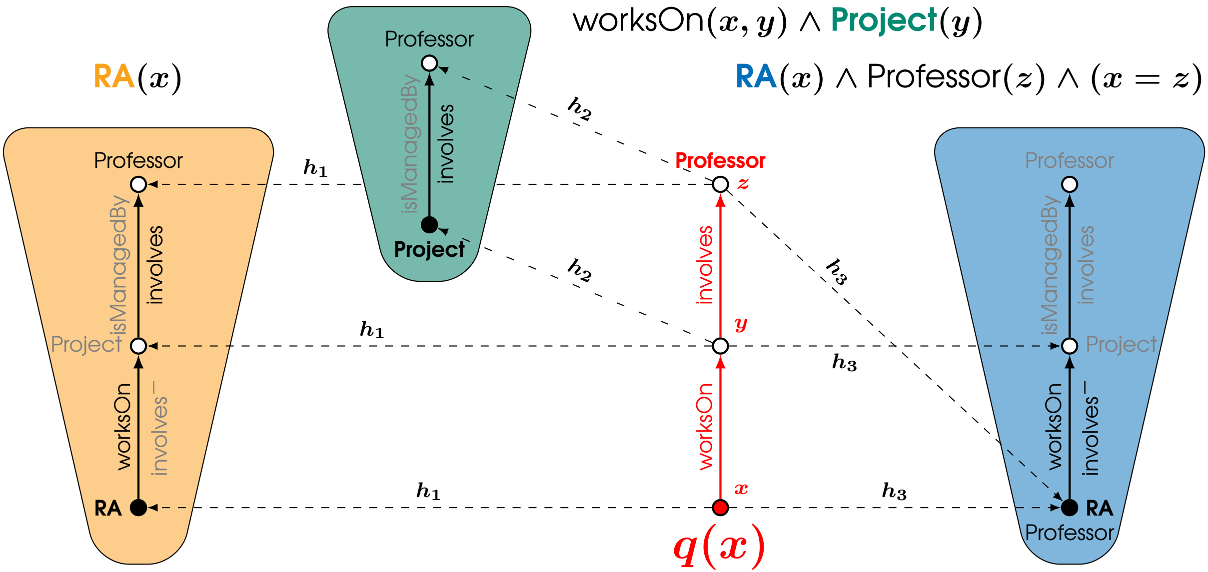

(x and z are the **roots** of the tree witness)

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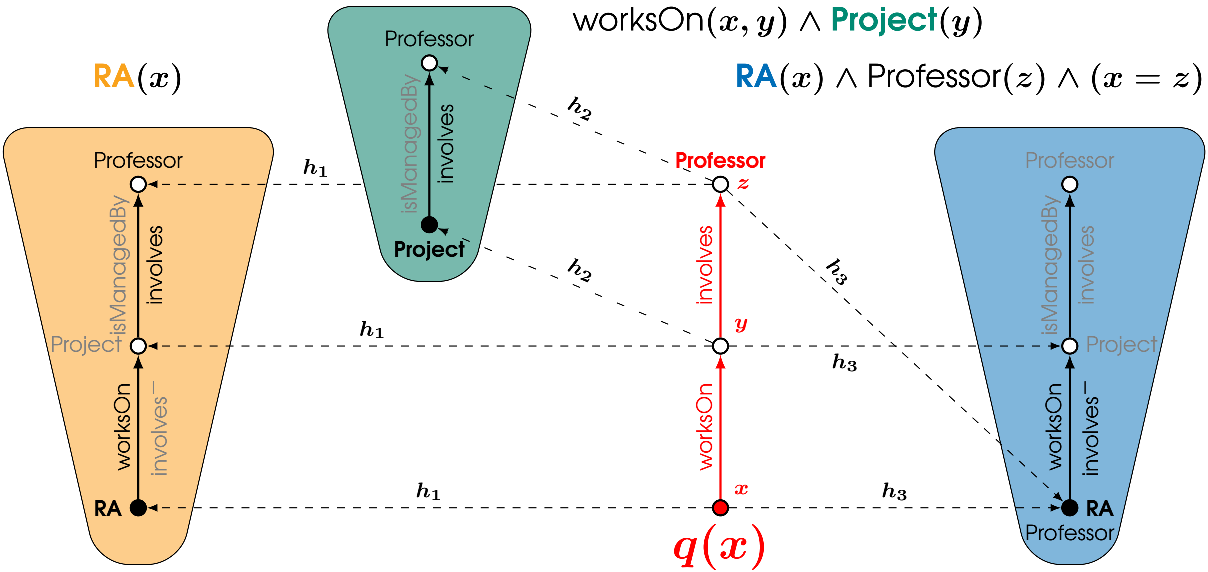


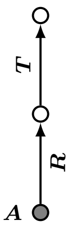
(x and z are the **roots** of the tree witness)

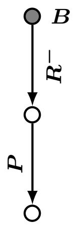
PE-rewriting (over H-complete ABoxes):

$$q'(x) \leftarrow \text{RA}(x) \lor (\text{worksOn}(x, y) \land \text{Project}(y)) \lor$$
$$(\text{RA}(x) \land \text{Professor}(z) \land (x = z)) \lor$$
$$(\text{worksOn}(x, y) \land \text{involves}(y, z) \land \text{Professor}(z))$$

Tree-Witness Rewriting

TBox $\mathcal{T}$: $\quad A \sqsubseteq \exists R, \quad \exists R^- \sqsubseteq \exists T, \quad B \sqsubseteq \exists R^-, \quad \exists R \sqsubseteq \exists S$





Tree-Witness Rewriting

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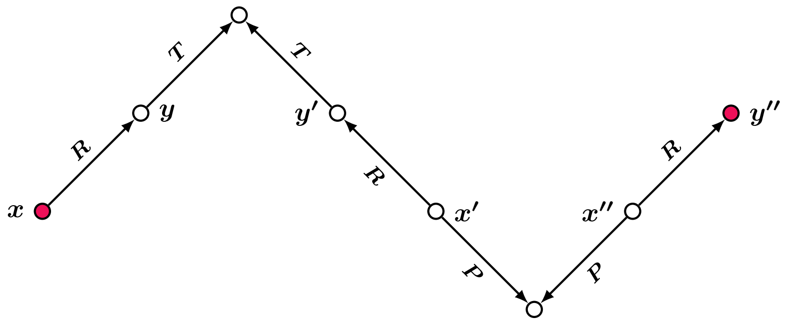

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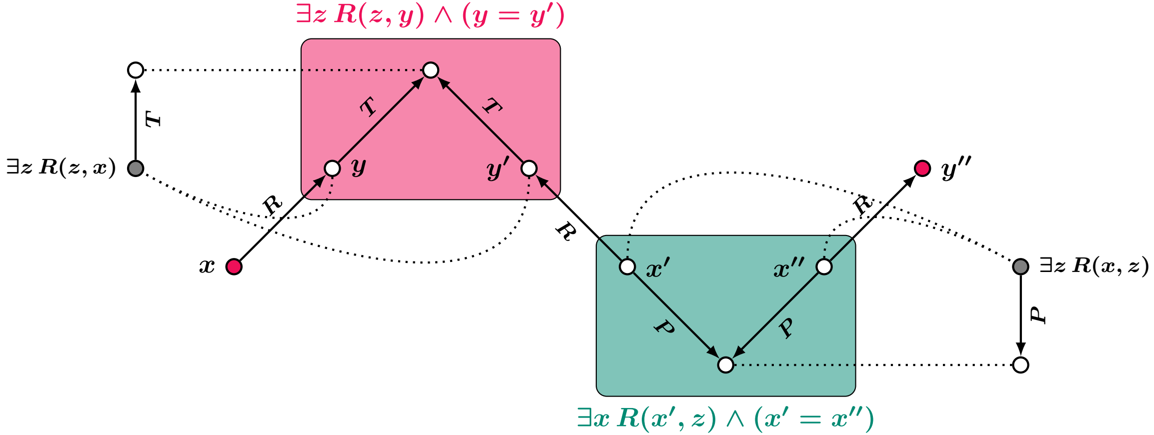

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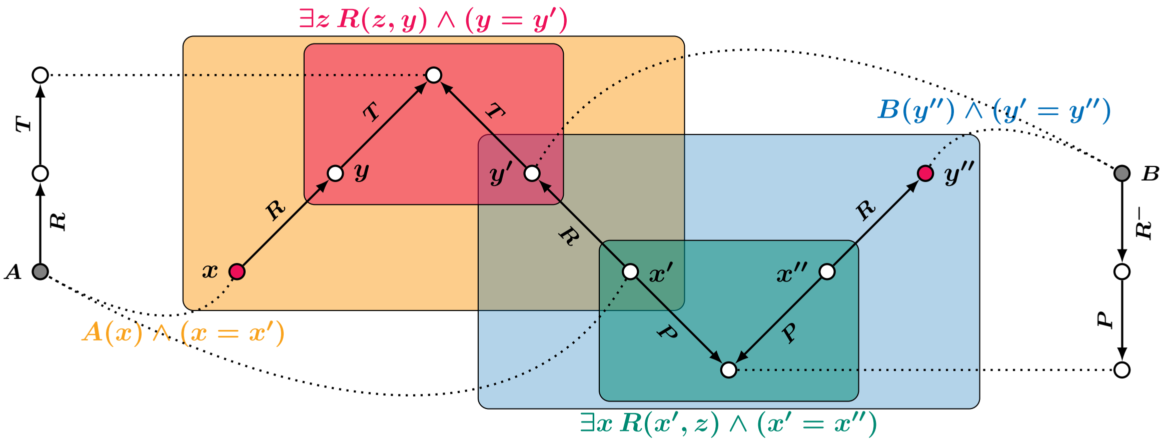

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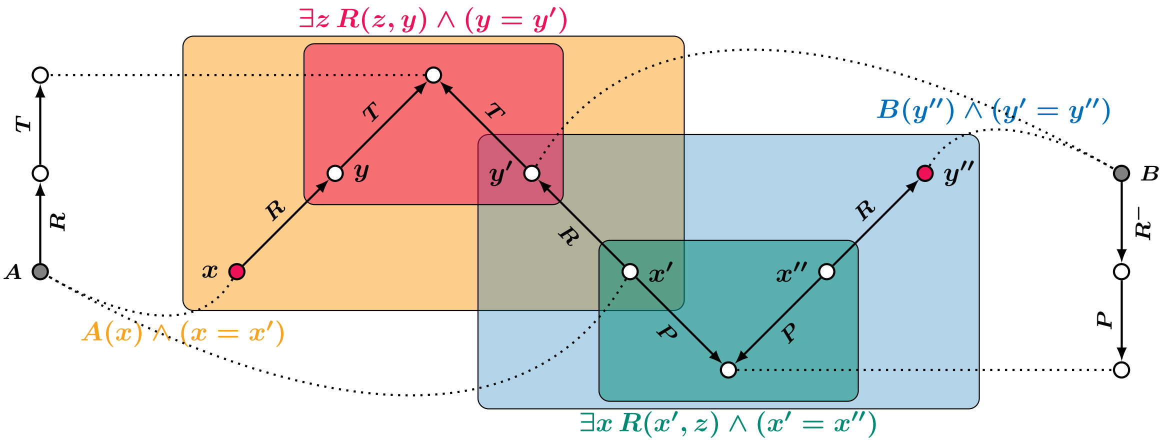


$$q_{\mathsf{tw}}(\vec{x}) \;=\; \bigvee_{\Theta \text{ independent set of tree witnesses}} \exists \vec{y}\, \Big(\bigwedge_{S(\vec{z}) \in q \setminus q_\Theta} S(\vec{z}) \;\wedge\; \bigwedge_{\mathfrak{t} \in \Theta} \mathsf{tw}_{\mathfrak{t}} \Big)$$

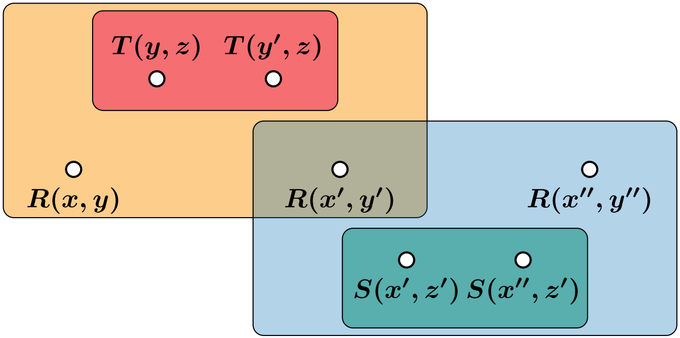
(Kikot, K & Zakharyaschev, 2012)

Rewritings as Boolean Functions

hypergraph H:

vertices = atoms of the query

hyperedges = tree witnesses



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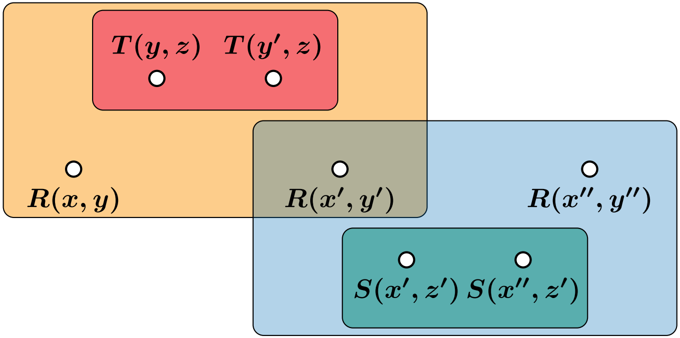

hypergraph function of $H = (V, E)$:

$$f_H = \bigvee_{\substack{X \subseteq E \\ X \text{ independent}}} \Big(\bigwedge_{v \in V \setminus V_X} p_v \wedge \bigwedge_{e \in X} p_e \Big)$$

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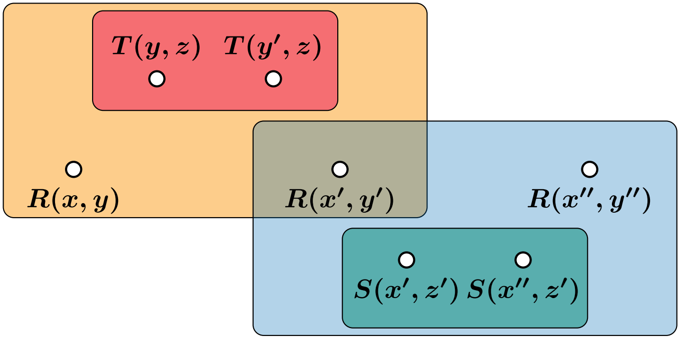

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(Kikot, K, Podolskii & Zakharyaschev, 2012) lower bounds from

circuit complexity

- **exponential** non-recursive datalog (and positive existential) rewritings
- **superpolynomial** first-order rewritings (unless NP $\subseteq$ P/poly)

Short Rewritings in Theory

if $q_{\mathfrak{t}_1} \cap q_{\mathfrak{t}_2} = \emptyset$ or $q_{\mathfrak{t}_1} \subseteq q_{\mathfrak{t}_2}$ or $q_{\mathfrak{t}_2} \subseteq q_{\mathfrak{t}_1}$, for each pair $\underbrace{\mathfrak{t}_1 \text{ and } \mathfrak{t}_2}_{\text{compatible}}$, then

$$q'_{\mathsf{tw}}(\vec{x}) = \bigwedge_{S(\vec{z}) \in q} \Big(S(\vec{z}) \lor \bigvee_{\mathfrak{t}\colon S(\vec{z}) \in q_{\mathfrak{t}}} \mathsf{tw}_{\mathfrak{t}} \Big)$$

is a **rewriting** (over H-complete ABoxes)

Short Rewritings in Theory

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is a **rewriting** (over H-complete ABoxes)

QL: replace $S(\vec{z})$ with $\bigvee_{\mathcal{T} \models S' \sqsubseteq S} S'(\vec{z})$

$\Longrightarrow$ **polynomial positive existential rewriting**

provided that the number of tree witnesses is **polynomial** and they are **compatible**

not the case in general!

Part 4

Practical OBDA with Ontop

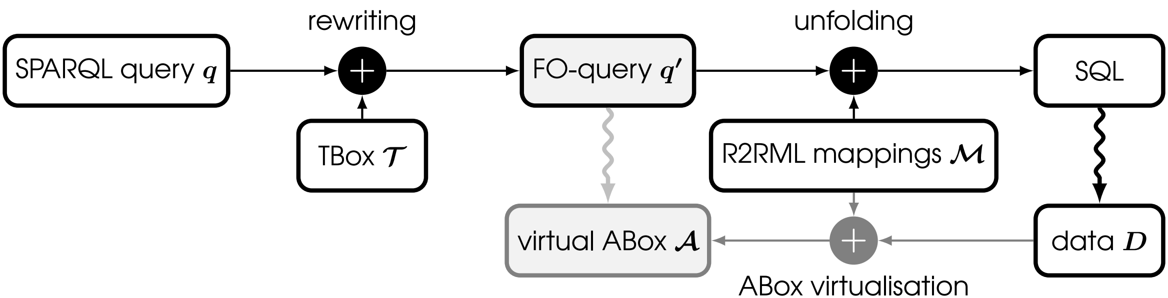
OBDA system Ontop

`http://ontop.inf.unibz.it`

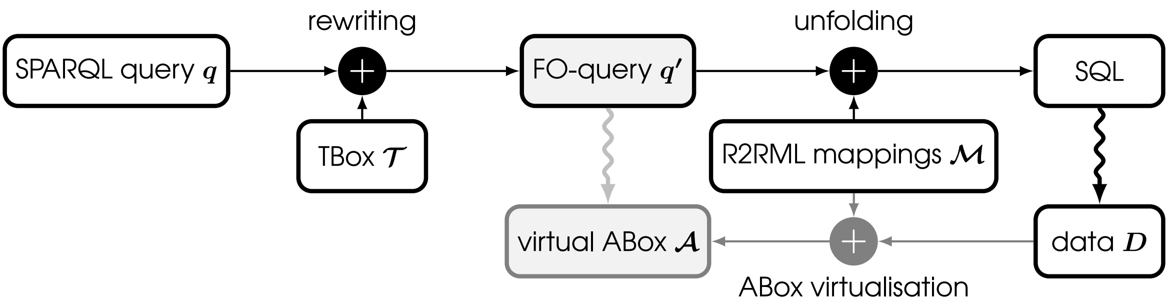
- implemented at the Free University of Bozen-Bolzano
  (Mariano Rodríguez-Muro, Martin Rezk, Guohui Xiao)
- open-source
- available as a plugin for Protégé 4 & 5, SPARQL end-point,
  OWL API and Sesame libraries

ontop framework

OBDA with Databases



OBDA with Databases



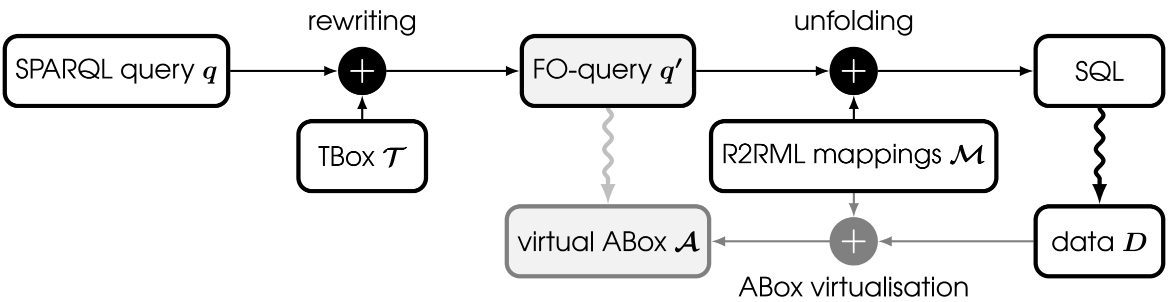
Why SQL rewritings are large:

(1) a large number of tree witnesses

(2) large concept/role hierarchies in OWL 2 QL ontology $\mathcal{T}$

(3) multiple definitions of the ontology terms in R2RML mappings $\mathcal{M}$

OBDA with Databases



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very few for real-world CQs/ontologies

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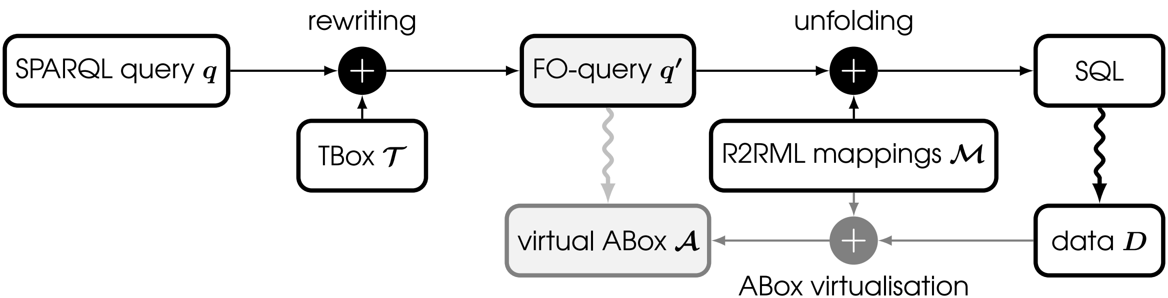
OBDA with Databases



Why SQL rewritings are large:

(1) a large number of tree witnesses

very few for real-world CQs/ontologies

(2) large concept/role hierarchies in OWL 2 QL ontology $\mathcal{T}$

many inclusions in $\mathcal{T}$ follow from Σ and $\mathcal{M}$

(3) multiple definitions of the ontology terms in R2RML mappings $\mathcal{M}$

Ontop Example

IMDb (simplified): `http://www.imdb.com/interfaces`

– database

title

movie ID	title	production year
$\mathbf{728}$	'Django Unchained'	$\mathbf{2012}$
...	...	...

castinfo

person ID	movie ID	person role
n$\mathbf{37}$	$\mathbf{728}$	$\mathbf{1}$
n$\mathbf{38}$	$\mathbf{728}$	$\mathbf{1}$
...	...	...

– dependencies

$$\forall m \, \big(\exists p, r \; \mathit{castinfo}(p, m, r) \to \exists t, y \; \mathit{title}(m, t, y)\big) \quad \text{(FK)}$$

$$\forall m \forall t_1 \forall t_2 \, \big(\exists y \; \mathit{title}(m, t_1, y) \land \exists y \; \mathit{title}(m, t_2, y) \to (t_1 = t_2)\big) \quad (\text{PK}_1)$$

$$\forall m \forall y_1 \forall y_2 \, \big(\exists t \; \mathit{title}(m, t, y_1) \land \exists t \; \mathit{title}(m, t, y_2) \to (y_1 = y_2)\big) \quad (\text{PK}_2)$$

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Movie Ontology MO http://www.movieontology.org

$\mathit{mo{:}Movie} \equiv \exists \mathit{mo{:}title}$, $\quad \mathit{mo{:}Movie} \sqsubseteq \exists \mathit{mo{:}year}$,

$\mathit{mo{:}Movie} \equiv \exists \mathit{mo{:}cast}$, $\quad \exists \mathit{mo{:}cast}^{-} \sqsubseteq \mathit{mo{:}Person}, \ldots$

Ontop Example

IMDb (simplified): http://www.imdb.com/interfaces

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movie ID	title	production year
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$$\textit{mo:Movie} \equiv \exists \textit{mo:title}, \qquad \textit{mo:Movie} \sqsubseteq \exists \textit{mo:year},$$

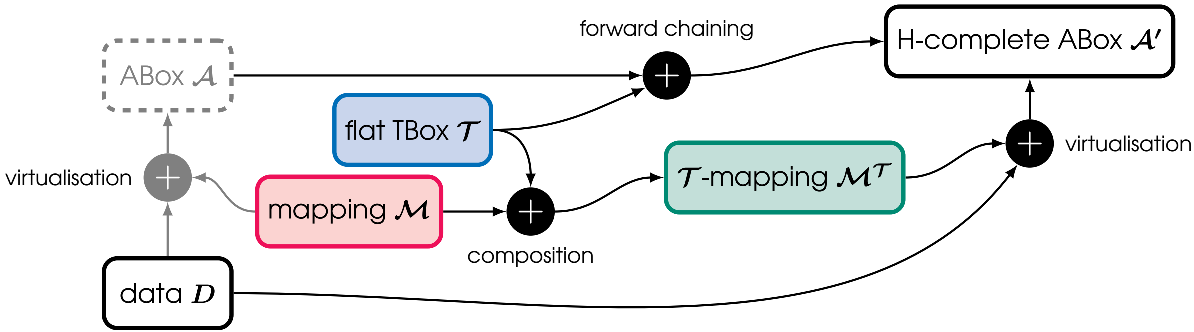
$$\textit{mo:Movie} \equiv \exists \textit{mo:cast}, \qquad \exists \textit{mo:cast}^- \sqsubseteq \textit{mo:Person}, \ldots$$

Mappings (created by the Ontop development team)

$$\textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y) \leftarrow \textit{title}(m, t, y) \qquad (\text{M}_1)$$

$$\textit{mo:cast}(m, p), \textit{mo:Person}(p) \leftarrow \textit{castinfo}(p, m, r) \qquad (\text{M}_2)$$

Ontop: T-mappings



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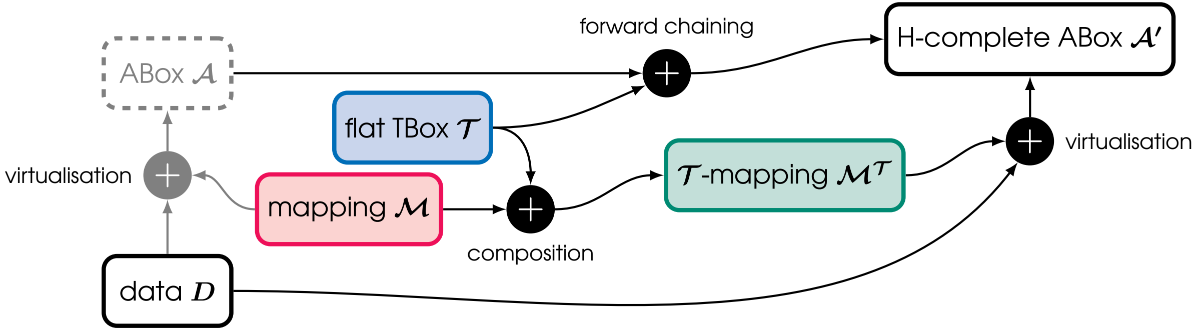
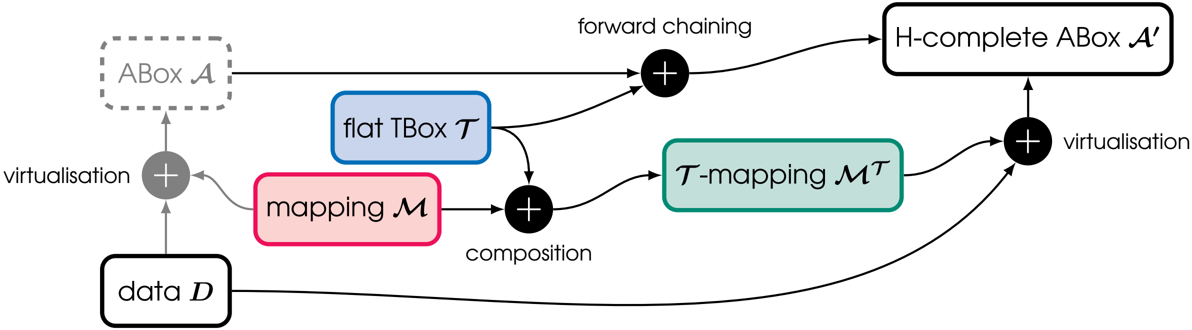


$\mathcal{T}$

$mo{:}Movie \equiv \exists mo{:}title$, $\quad mo{:}Movie \sqsubseteq \exists mo{:}year$,
$mo{:}Movie \equiv \exists mo{:}cast$, $\quad \exists mo{:}cast^{-} \sqsubseteq mo{:}Person$

Ontop: T-mappings



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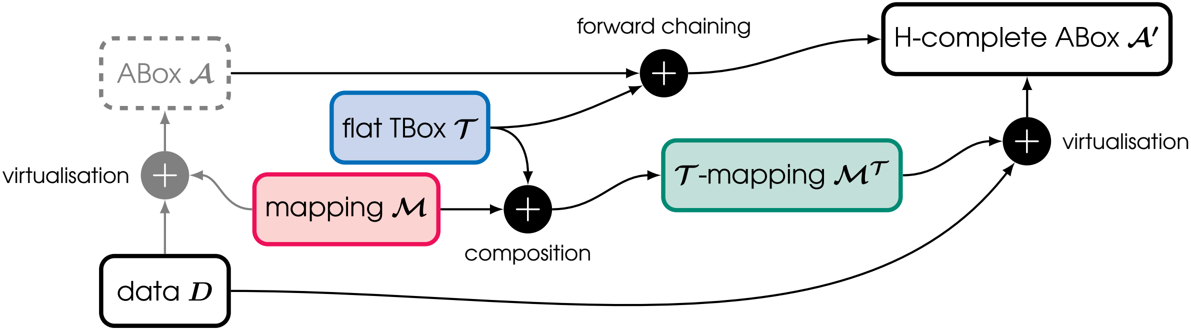
$\mathcal{M}$

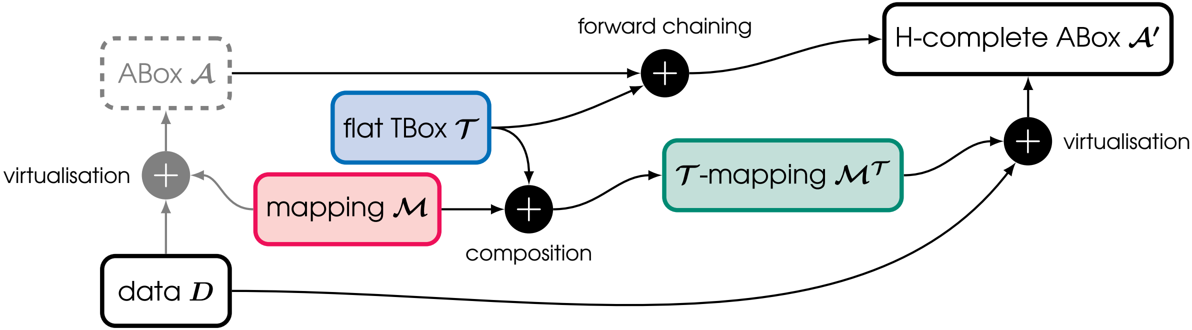
$$\textit{mo:Movie}(\boldsymbol{m}), \textit{mo:title}(\boldsymbol{m}, \boldsymbol{t}), \textit{mo:year}(\boldsymbol{m}, \boldsymbol{y}) \leftarrow \textit{title}(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{y}) \qquad (\mathrm{M}_1)$$
$$\textit{mo:cast}(\boldsymbol{m}, \boldsymbol{p}), \textit{mo:Person}(\boldsymbol{p}) \leftarrow \textit{castinfo}(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r}) \qquad (\mathrm{M}_2)$$

Ontop: T-mappings



$\mathcal{T}$	*mo:Movie* $\equiv \exists$*mo:title*, *mo:Movie* $\sqsubseteq \exists$*mo:year*, *mo:Movie* $\equiv \exists$*mo:cast*, $\exists$*mo:cast*$^-$ $\sqsubseteq$ *mo:Person*
$\mathcal{M}$	*mo:Movie*$(\boldsymbol{m})$, *mo:title*$(\boldsymbol{m}, \boldsymbol{t})$, *mo:year*$(\boldsymbol{m}, \boldsymbol{y}) \leftarrow$ *title*$(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{y})$ (M_1) *mo:cast*$(\boldsymbol{m}, \boldsymbol{p})$, *mo:Person*$(\boldsymbol{p}) \leftarrow$ *castinfo*$(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r})$ (M_2)
$\mathcal{M}^{\mathcal{T}}$	*mo:Movie*$(\boldsymbol{m}) \leftarrow$ *title*$(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{y})$ by (M_1) *mo:Movie*$(\boldsymbol{m}) \leftarrow$ *castinfo*$(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r})$ by (M_2) + $\exists$*mo:cast* $\sqsubseteq$ *mo:Movie*

Ontop: T-mappings



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mo:Movie $\equiv \exists$*mo:title*, *mo:Movie* $\sqsubseteq \exists$*mo:year*,
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$\mathcal{M}$

mo:Movie(m), *mo:title*(m, t), *mo:year*$(m, y) \leftarrow$ *title*(m, t, y) (M_1)
mo:cast(m, p), *mo:Person*$(p) \leftarrow$ *castinfo*(p, m, r) (M_2)

$\mathcal{M}^{\mathcal{T}}$

mo:Movie$(m) \leftarrow$ *title*(m, t, y) by (M_1)
~~*mo:Movie*$(m) \leftarrow$ *castinfo*(p, m, r)~~ by (M_2) + $\exists$*mo:cast* $\sqsubseteq$ *mo:Movie*

redundant by (FK)

$\forall m\ (\exists p, r$ ***castinfo***$(p, m, r) \rightarrow \exists t, y$ ***title***$(m, t, y))$

Optimising T-mappings

- using foreign keys (inclusion dependencies)

Optimising T-mappings

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- using disjunction

$\mathcal{T}$

mo:Actor $\sqsubseteq$ *mo:Artist*, *mo:Artist* $\sqsubseteq$ *mo:Person*,
mo:Director $\sqsubseteq$ *mo:Person*, *mo:Editor* $\sqsubseteq$ *mo:Person*, ...

$\mathcal{M}$

mo:Actor$(\boldsymbol{p}) \leftarrow$ *castinfo*$(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r}), (\boldsymbol{r} = \mathbf{1})$ (M_1)

...

mo:Editor$(\boldsymbol{p}) \leftarrow$ *castinfo*$(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r}), (\boldsymbol{r} = \mathbf{6})$ (M_6)

Optimising T-mappings

- using foreign keys (inclusion dependencies)

- using disjunction

$\mathcal{T}$

mo:Actor $\sqsubseteq$ *mo:Artist*, *mo:Artist* $\sqsubseteq$ *mo:Person*,
mo:Director $\sqsubseteq$ *mo:Person*, *mo:Editor* $\sqsubseteq$ *mo:Person*, ...

$\mathcal{M}$

mo:Actor$(p) \leftarrow$ *castinfo*$(p, m, r), (r = 1)$ (M$_1$)
...
mo:Editor$(p) \leftarrow$ *castinfo*$(p, m, r), (r = 6)$ (M$_6$)

$\mathcal{M}^{\mathcal{T}}$

mo:Person$(p) \leftarrow$ *castinfo*$(p, m, r), ((r = 1) \lor \cdots \lor (r = 6))$

Unfolding with Semantic Query Optimisation

Query

$$\boldsymbol{q}(\boldsymbol{t}, \boldsymbol{y}) \leftarrow \textit{mo:Movie}(\boldsymbol{m}), \textit{mo:title}(\boldsymbol{m}, \boldsymbol{t}), \textit{mo:year}(\boldsymbol{m}, \boldsymbol{y}), (\boldsymbol{y} > \mathbf{2010})$$

Unfolding with Semantic Query Optimisation

Query

$$q(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

Rewriting

$$q'(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

Unfolding with Semantic Query Optimisation

Query

$$q(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

Rewriting

$$q'(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

$\mathcal{M}$

$\textit{mo:Movie}(m) \leftarrow \textit{title}(m, t, y)$	(M$_1$)
$\textit{mo:title}(m, t) \leftarrow \textit{title}(m, t, y)$	(M$_2$)
$\textit{mo:year}(m, y) \leftarrow \textit{title}(m, t, y)$	(M$_3$)

Unfolding with Semantic Query Optimisation

Query

$$q(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

Rewriting

$$q'(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

$\mathcal{M}$

$\textit{mo:Movie}(m) \leftarrow \textit{title}(m, t, y)$	(M_1)
$\textit{mo:title}(m, t) \leftarrow \textit{title}(m, t, y)$	(M_2)
$\textit{mo:year}(m, y) \leftarrow \textit{title}(m, t, y)$	(M_3)

Unfolding

$$q^*(t, y) \leftarrow \textit{title}(m, t_0, y_0), \textit{title}(m, t, y_1), \textit{title}(m, t_2, y), (y > 2010)$$

Unfolding with Semantic Query Optimisation

Query

$$q(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

Rewriting

$$q'(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

$\mathcal{M}$

$$\textit{mo:Movie}(m) \leftarrow \textit{title}(m, t, y) \quad (\text{M}_1)$$
$$\textit{mo:title}(m, t) \leftarrow \textit{title}(m, t, y) \quad (\text{M}_2)$$
$$\textit{mo:year}(m, y) \leftarrow \textit{title}(m, t, y) \quad (\text{M}_3)$$

Unfolding

$$q^*(t, y) \leftarrow \textit{title}(m, t_0, y_0), \textit{title}(m, t, y_1), \textit{title}(m, t_2, y), (y > 2010)$$

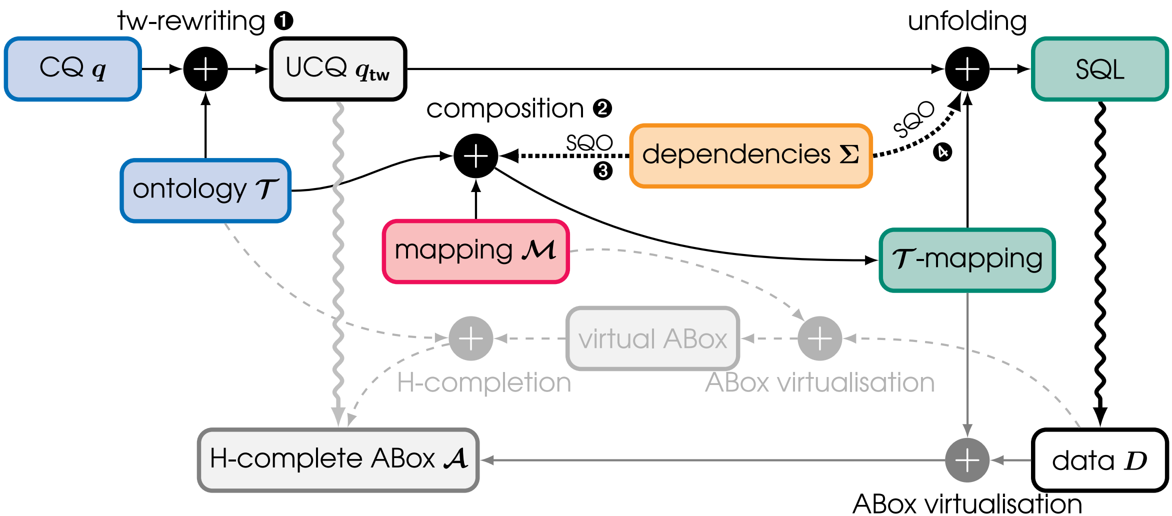
primary keys

$$\forall m \forall t_1 \forall t_2 \left(\exists y\, \textit{title}(m, t_1, y) \land \exists y\, \textit{title}(m, t_2, y) \to (t_1 = t_2)\right) \quad (\text{PK}_1)$$
$$\forall m \forall y_1 \forall y_2 \left(\exists t\, \textit{title}(m, t, y_1) \land \exists t\, \textit{title}(m, t, y_2) \to (y_1 = y_2)\right) \quad (\text{PK}_2)$$

Semantic Query Optimisation

$$q^\dagger(t, y) \leftarrow \textit{title}(m, t, y), (y > 2010)$$

Practical OBDA with Ontop



❶ tree-witness rewriting q_{tw} over H-complete ABoxes (no concept/role hierarchies)

❷ $\mathcal{T}$-mapping = system mapping $\mathcal{M} + \mathcal{T}$ makes virtual ABoxes H-complete

❸ $\mathcal{T}$-mapping is simplified using SQO and SQL features

constructed and optimised for $\mathcal{T}$ and Σ only once

❹ unfolding uses SQO to produce small and efficient SQL queries

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